EXERCISE - 01 CHECK YOUR GRASP

2. Since product of any r consecutive integers (क्रमागत पूर्णांक) is divisible by r! and not divisible by r+1!.

So given product of 4 consecutive integers is divisible by 4! or 24.

3. Let p(n) = $n^2 + n = n(n + 1)$ is an odd integer since the product of two consecutive integers is always even. So hear principle of induction (आगमन सिद्धान्त) is not aplicable.

5. Since $x^n + y^n$ is divisible $(x + y)$ if n is odd.

Here $2n - 1$ is odd $\forall$ $n \in N$.

7. Here $T_n = n(n + 1)^2$

$\therefore \quad S_n = \Sigma T_n = \Sigma n^3 + 2\Sigma n^2 + \Sigma n$

$$= \frac{n^2(n+1)^2}{4} + 2.\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{1}{12} n(n + 1)(n + 2)(3n + 5)$$

8. Let three consecutive natural numbers are n, n+1, n+2, $P(n) = (n)^3 + (n+1)^3 + (n+2)^3$

$P(1)=1^3 + 2^3 + 3^3=36$, which is divisible by 2 and 9

$P(2) = (2)^3 + (3)^3 + (4)^3 = 99$, which is divisible by 9 (not by 2).

Hence P(n) is divisible 9 $\forall$ $n \in N$.

9. Let $P(n) = 11^{n+2} + 12^{2n+1}$

$P(1) = 11^3 + 12^3 = 23 \quad 133$, which is divisible by 133 but not by 113 and 123.

10. Let $p(n) = 3^{4n+2} + 5^{2n+1}$

Here $P(1) = 3^6 + 5^3 = 9^3 + 5^3 = 14 \quad 61$

Which is multiple of 14 but not of 16, 18 and 20.

12. Let n is a positive integer.

$P(n) = n^3 - n$

P(1) = 0, which is divisible by for all $n \in N$

P(2) = 6, which is divisible by 6 (not by 4 and 9)

15. Let $P(n) = \frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$

$$P(1) = \frac{1}{7} + \frac{1}{5} + \frac{2}{3} - \frac{1}{105} = 1 \text{ (integer)}$$

$$P(2) = 2^4\left(\frac{8}{7} + \frac{2}{5} + \frac{1}{3}\right) - \frac{2}{105} = 15 \text{ (integer) etc.}$$

Hence P(n) is an integer.

18. n^{th} term of the given series

$$T_n = \frac{\frac{n}{2}.\frac{n+1}{2}}{\Sigma n^3} = \frac{\frac{1}{4}n(n+1)}{\frac{1}{4}n^2(n+1)^2} = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$\therefore S_n = \left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + --- + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right]$$

$$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

19. Let $P(n) = 7^{2n} - 48n - 1$

$P(1) = 7^2 - 48.1 - 1 = 0$,

which is divisible by for all $n \in N$

$P(2) = 7^4 - 48.2 - 1 = 2304$,

which is divisible by 2304 not by 25, 26 and 1234.

20. Let n^{th} term of the series is T_n and

$S_n = 4 + 14 + 30 + 52 + 80 + 114 + ---- + T_n$ ..(i)

$S_n = \quad 4 + 14 + 30 + 52 + 80 + ---- + T_n$ ..(ii)

Subtract (ii) from (i)

$0=(4 + 10 + 16 + 22 + 28 + 34 + ---- \text{ n terms}) - T_n$

$T_n = 4 + 10 + 16 + 22 + ----$ n terms

$$= \frac{n}{2}[2 \quad 4 + (n - 1)6] = n(3n + 1) = 3n^2 + n$$

21. Let $P(n) = 10^n + 3.4^{n+2} + \lambda$ is divisible by 9 $\forall$ $n \in N$

$P(1) = 10 + 3.4^3 + \lambda = 202 + \lambda = 207 + (\lambda-5)$

Which is divisible by 9 if $\lambda = 5$

22. $T_n = 1 + a + a^2 + ------- + a^{n-1} = \frac{1 - a^n}{1 - a}$

$$S_n = \Sigma T_n = \frac{1}{(1-a)}[\Sigma 1 - \Sigma a^n]$$

$$= \frac{1}{(1-a)}[n - (a + a^2 + a^3 + ---- a^n)]$$

$$= \frac{1}{(1-a)}\left[n - \frac{a(1-a^n)}{(1-a)}\right] = \frac{n}{1-a} - \frac{a(1-a^n)}{(1-a)^2}$$

25. Let $P(n) = \cos\theta \,.\, \cos 2\theta \,.\, \cos 4\theta$ ----- $\cos 2^{n-1}\theta$

$$P(1) = \cos\theta = \frac{2\sin\theta\cos\theta}{2\sin\theta} = \frac{\sin 2\theta}{2\sin\theta}$$

$$P(2) = \cos\theta\cos 2\theta = \frac{2(2\sin\theta\cos\theta)\cos 2\theta}{4\sin\theta}$$

$$= \frac{2\sin 2\theta\cos 2\theta}{4\sin\theta}$$

$$= \frac{\sin 4\theta}{4\sin\theta} = \frac{\sin 2^2\theta}{2^2\sin\theta}$$

Clearly, $P(n) = \dfrac{\sin 2^n\theta}{2^n\sin\theta}$

28. Let $P(n)$: $3^{n+1} < 4^n$

$P(1)$: $3^2 < 4$ which is false

$P(2)$: $3^3 < 4^2$ which is false

$P(3)$: $3^4 < 4^3$ which is false

$P(4)$: $3^5 < 4^4$ which is true

33. Let $P(n)$: $n^p - n$

when $p = 2$

$P(n) = n^2 - n$

$P(1) = 0$ which is divisible all $n \in N$

$P(2) = 2$ which is divisible by 2

$P(3) = 6$ which is divisible by 2

Hence P(n) is divisible by 2 when n is greater than 1.

35. By Theorem-II

EXERCISE - 02 PREVIOUS YEAR QUESTION

1. $S(K) = 1 + 3 + 5 + \ldots\ldots. + (2K - 1) = 3 + K^2$

S(1) is not true

let S(K) is true then

$S(K + 1) = 1 + 3 + 5 + \ldots.. + (2K - 1) + (2K + 1)$

$= S(K) + (2K + 1)$

$= 3 + K^2 + 2K + 1 = 3 + (K + 1)^2$

Hence $S(K) \Rightarrow S(K + 1)$

2. Since $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \text{----}$

n terms $= \dfrac{n(n+1)^2}{2}$, when n is even

When n is odd the n^{th} term of series will be n^2 in this case, $(n - 1)$ is even

so for finding sum of first $(n - 1)$ terms of the series, we replacing n by $(n - 1)$ in the given formula.

So sum of first $(n - 1)$ terms $= \dfrac{(n-1)n^2}{2}$

Hence sum of n terms of the series

= (the sum of $(n - 1)$ terms + the n^{th} term)

$$= \frac{(n-1)n^2}{2} + n^2 = \frac{(n+1)n^2}{2}$$

3. $$A^2 = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$A^3 = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}$$

$$\therefore \quad A^n = \begin{bmatrix}1 & 0\\n & 1\end{bmatrix}$$

Now $$nA-(n-1)I = \begin{bmatrix}n & 0\\n & n\end{bmatrix} - \begin{bmatrix}n-1 & 0\\0 & n-1\end{bmatrix} = \begin{bmatrix}1 & 0\\n & 1\end{bmatrix} = A^n$$

4. $\because \sqrt{n(n+1)} < \sqrt{(n+1)(n+1)}$

i.e. $\sqrt{n(n+1)} < n + 1 \quad \forall\, n \in N$

Hence statement-2 is true.

For $n = 2$ given result is true.

let it is true for $n = K \in N$, $K \geq 2$ then

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots.. + \frac{1}{\sqrt{K}} > \sqrt{K}$$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots.. + \frac{1}{\sqrt{K}} + \frac{1}{\sqrt{K+1}} > \sqrt{K} + \frac{1}{\sqrt{K+1}}$$

$$= \frac{\sqrt{K(K+1)} + 1}{\sqrt{K+1}} > \frac{\sqrt{KK} + 1}{\sqrt{K+1}} = \sqrt{K+1}$$

($\because$ by statement-2 $\sqrt{n(n+1)} < n+1 \Rightarrow \sqrt{n} < \sqrt{n+1}$)

$$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots.. + \frac{1}{\sqrt{K+1}} > \sqrt{K+1}$$

Hence statement-1 is true for every natural number $n \geq 2$.