EXERCISE - 01 CHECK YOUR GRASP

5. $\because (p \to \sim p) \wedge (\sim p \to p)$

$\equiv (\sim p \vee \sim p) \wedge (p \vee p)$ $(\because p \to q \equiv \sim p \vee q)$

$\equiv \sim p \wedge p$ $(\because p \vee p \equiv p)$

$\equiv c$ c is a contradiction

8. $\because \sim q \to \sim p \equiv \sim(\sim q) \vee \sim p$ $(\because p \to q \equiv \sim p \vee q)$

$\equiv q \vee \sim p$

$\equiv \sim p \vee q$ (by commutative law)

$\equiv p \to q$ $(\because p \to q = \sim p \vee q)$

Hence $p \to q \equiv \sim q \to \sim p$

12. $\because$ $(p \wedge q) \to p$ is false

$\Rightarrow$ $(p \wedge q)$ is true and p is false

which is not possible

so $(p \wedge q) \to p$ is always true i.e. it is a tautology.

14. $\because$ $p \to (q \vee r)$ is false

$\Rightarrow$ p is true and $(q \vee r)$ is false

$\Rightarrow$ p is true, q and r both are false

i.e. $p \to (q \vee r)$ is false when truth values of p, q, r are T, F, F resp. otherwise it is true.

15. $\because$ $(p \wedge q) \wedge \sim (p \vee q)$ is true

$\Rightarrow$ $(p \wedge q)$ and $\sim(p \vee q)$ both are true

which is not possible

So $(p \wedge q) \wedge \sim (p \vee q)$ is always false i.e. it is a contradiction.

16. Let p, q, r three statement defined as

p : a number N is divisible by 15

q : number N is divisible by 5

r : number N is divisible by 3

Here given statement is $p \to (q \vee r)$

Here negative of above statement is

$\sim(p \to (q \vee r)) \equiv p \wedge (\sim(q \vee r)$

$\equiv p \wedge (\sim q \wedge \sim r)$

i.e. A number is divisible by 15 and it is not divisible by 5 and 3.

19. $\because$ $(p \wedge q) \vee (q \wedge r)$ is false

$\Rightarrow$ $(p \wedge q)$ and $(q \wedge r)$ both are false

$\Rightarrow$ p and r both are false or q is false.

otherwise $(p \wedge q) \vee (q \wedge r)$ is true

20. $\because$ $(\sim p \vee \sim q) \vee (p \vee \sim q)$

$\equiv (\sim p \vee \sim q) \vee (\sim q \vee p)$ (by commutative law)

$\equiv \sim p \vee [\sim q \vee (\sim q \vee p)]$ (by Associative law)

$\equiv \sim p \vee [(\sim q \vee \sim q) \vee p]$ (by Associative law)

$\equiv \sim p \vee (\sim q \vee p)$ $(\because p \vee p = p)$

$\equiv \sim p \vee (p \vee \sim q)$ (by commutative law)

$\equiv (\sim p \vee p) \vee \sim q$ (by Associative law)

$\equiv t \vee \sim q \equiv t$ t is a tautology

Hence $(\sim p \vee \sim q) \vee (p \vee \sim q)$ is a tautology.

24. When p and q both are true then

$\sim(p \to q)$ and $(\sim p \vee \sim q)$ both are false

i.e. $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$ is true

when p and q both are false then

$\sim(p \to q)$ is false and $(\sim p \vee \sim q)$ is true

i.e. $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$ is false

Hence $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$ is neither tautology nor contradiction.

28. Let $S(p, q) \equiv (p \vee \sim q) \wedge \sim p$

$\Rightarrow$ $S(\sim p, \sim q) \equiv (\sim p \vee q) \wedge p$

Now $S^*(\sim p, \sim q) \equiv (\sim p \wedge q) \vee p$

and $\sim S(p, q) \equiv \sim[(p \vee \sim q) \wedge \sim p] \equiv \sim(p \vee \sim q) \vee p$

$\equiv (\sim p \wedge q) \vee p$

Hence $S^*(\sim p, \sim q) \equiv \sim S(p, q)$

32. $\because$ $[(p \wedge p) \to q] \to p \equiv (p \to q) \to p$ $(\because p \wedge p \equiv p)$

when p is false and q is true (or false) then

$(p \to q)$ is true i.e. $(p \to q) \to p$ is false

Hence $[(p \wedge p) \to q] \to p$ is not a tautology.

34. $\because$ $(p \vee \sim r) \to (q \wedge r)$ is false

$\Rightarrow$ $(p \vee \sim r)$ is true, and $(q \wedge r)$ is false

Here $(q \wedge r)$ is false and q is true (given)

$\Rightarrow$ r is false

again r is false and $(p \vee \sim r)$ is true

$\Rightarrow$ p may be true or false.

1. $\because$ $p \to (q \to p)$ is false

$\Rightarrow$ p is true and $(q \to p)$ is false. which is not possible.

So $p \to (q \to p)$ is always true i.e. it is a tautology.

Again $p \to (p \vee q)$ is false

p is true and $(p \vee q)$ is false. Which is not possible.

So $p \to (p \vee q)$ is always true i.e. it is a tautology.

Hence $p \to (q \to p) \equiv p \to (p \vee q)$

2. Given $r : \sim p \leftrightarrow q$

statement-1 $r \equiv q \vee p$

statement-2 $r \equiv (p \leftrightarrow \sim q)$

p	q	~ p	~ q	(~ p ↔ q)	q ∨ p	(p ↔ ~ q)
T	T	F	F	F	T	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	F	F

Hence Statement-1 is false and Statement-2 is true.

3. statement-1 : $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

statement-2 : $\sim(p \leftrightarrow \sim q)$ is a tautology.

p	q	~ q	(p ↔ q)	(p ↔ ~ q)	~ (p ↔ ~ q)
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

Hence statement-1 is true, statement-2 is false.

4. Given $S \subseteq R$ and

p=There is a rational number $x \in S$ such that $x>0$

then $\sim p$: Any rational number $x \in S$ such that $x \not> 0$

i.e. $\sim p$: Every rational number $x \in S$ satisfy $x \leq 0$

5. Given Statement :

$(p \wedge \sim r) \Leftrightarrow q$

Negations of $p \Leftrightarrow q$ are

$\sim (p \Leftrightarrow q)$, $\sim (q \Leftrightarrow p)$,

$\sim p \Leftrightarrow q$ and $\sim q \Leftrightarrow p$

Hence negations of given statement are $\sim(q \Leftrightarrow (p \wedge \sim r))$

and $\sim (p \wedge \sim r) \Leftrightarrow q$

6. $[p \wedge (p \to q)] \to q$

$[p \wedge (\sim p \vee q)] \to q$

$[(p \wedge \sim p) \vee (p \wedge q)] \to q$

$[c \vee (p \wedge q)] \to q$ $\left\{\begin{array}{l} p \wedge \sim p \equiv c \equiv \text{contradiction} \\ \because c \vee p \equiv p \end{array}\right.$

$\Rightarrow (p \wedge q) \to q$

$\Rightarrow \sim(p \wedge q) \vee q$

$\Rightarrow (\sim p \vee \sim q) \vee q$

$\Rightarrow \sim p \vee (q \vee \sim q)$

$\Rightarrow \sim p \vee (t) \equiv$ tautology