3D-COORDINATE GEOMETRY

EXERCISE - 01 — CHECK YOUR GRASP

5. Equation of plane containing L_1 and parallel to L_2 is $\begin{vmatrix} x-2 & y-1 & z+1 \\ 1 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow \quad 2x - 3y - z = 2$

distance from origin $= \frac{2}{\sqrt{14}} = \sqrt{\frac{2}{7}}$

6. Equation of plane is $\vec{r}.\vec{n} = \frac{q}{|\vec{n}|}$

for intercept on x-axis take dot product with $\tilde{i}$

$\Rightarrow$ intercept on x-axis = $\frac{q}{\tilde{i}.\vec{n}}$

7. From P(f, g, h) the foot of perpendicular on plane yz = (0, g, h),

similarly from P(f, g, h) perpendicular to z x = (f,0,h)

Equation of plane is

$\begin{vmatrix} x & y & z \\ f & 0 & h \\ 0 & g & h \end{vmatrix} = 0 \Rightarrow \frac{x}{f} + \frac{y}{g} - \frac{z}{h} = 0$

12. Let the tetrahedron cut x-axis, y-axis and z-axis at a, b & c respectively.

volume = $\frac{1}{6}[a\tilde{i}\ b\tilde{j}\ c\tilde{k}]$ (Given)

Then $\frac{1}{6}(abc) = 64K^3$ .....(1)

Let centroid be (x_1, y_1, z_1)

$\therefore x_1 = \frac{a}{4},\ y_1 = \frac{b}{4},\ z_1 = \frac{c}{4}$

put in (1) wet get

$x_1y_1z_1 = 6K^3$

$\therefore$ Locus is $xyz = 6K^3$

The required locus is $xyz = 6K^3$

13. $\vec{r}.\vec{n} = d$ ....(1)

$\vec{r} = \vec{r}_0 + t\vec{n}$ ....(2)

from (1) and (2)

$(\vec{r}_0 + t\vec{n}).\vec{n} = d \quad \Rightarrow \quad t = \frac{d - \vec{r}_0.\vec{n}}{\vec{n}^2}$

substitute the value of 't' in (2)

$r = \vec{r}_0 + \left(\frac{d - \vec{r}_0.\vec{n}}{\vec{n}^2}\right)\vec{n}$

14. Equation of plane containing

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and point (0, 7, – 7) is

$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ -1 & -4 & 5 \end{vmatrix} = 0$

By solving we get

$x + y + z = 0$

EXERCISE - 02 — BRAIN TEASERS

1. $\vec{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k} + \lambda(\tilde{i} + \tilde{j} + \sqrt{2}\tilde{k})$

$\cos\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 30°, \cos\beta = \frac{\sqrt{3}}{2} \Rightarrow \beta = 30°,$

$\cos\gamma = \frac{\sqrt{2}}{2} \Rightarrow \gamma = 45$

By putting the values check options

4. Let any point on line $\frac{x-1}{2} = \frac{y+1}{-3} = z = \lambda$

be $(1 + 2\lambda, -1 - 3\lambda, \lambda)$

$4\sqrt{14} = \sqrt{(1+2\lambda-1)^2 + (-1-3\lambda+1)^2 + \lambda^2}$

$4\sqrt{14} = \sqrt{4\lambda^2 + 9\lambda^2 + \lambda^2}$

$\Rightarrow |\lambda| = 4 \quad \Rightarrow \lambda = \pm 4$

$\therefore$Points (9, – 13, 4) and (– 7, 11, – 4)

5. The vector parallel to line of intersection of planes

is $\lambda\begin{vmatrix} i & j & k \\ 6 & 4 & -5 \\ 1 & -5 & 2 \end{vmatrix} = -\lambda(17\tilde{i} + 17\tilde{j} + 34\tilde{k})$

$= \lambda'(\tilde{i} + \tilde{j} + 2\tilde{k})$ (λ' is scalar)

Now angle between the lines

$\cos\theta = \frac{\lambda'(\tilde{i} + \tilde{j} + 2\tilde{k}).(2\tilde{i} - \tilde{j} + \tilde{k})}{\lambda'\sqrt{6} \times \sqrt{6}} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

8. Equation of bisector of plane

$\frac{2x - y + 2z + 3}{\sqrt{2^2 + 1^2 + 2^2}} = \pm\frac{3x - 2y + 6z + 8}{\sqrt{9 + 4 + 36}}$

$\Rightarrow \frac{2x - y + 2z + 3}{3} = \pm\frac{(3x - 2y + 6z + 8)}{7}$

$\Rightarrow 14x - 7y + 14z + 21 = \pm(9x - 6y + 18z + 24)$

$\Rightarrow 5x - y - 4z = 3$ and

$23x - 13y + 32z + 45 = 0$

9. Let normal vector n_1 perpendicular to plane determining $\tilde{i}, \tilde{j}+\tilde{k}$ is

$n_1 = \tilde{i} \times (\tilde{i}+\tilde{j}) = \tilde{k}$

similarly $n_2 = (\tilde{i}-\tilde{j}) \times (\tilde{i}-\tilde{k}) = \tilde{i}+\tilde{j}+\tilde{k}$

Now vector parallel to intersection of plane $= \vec{n}_2 \times \vec{n}_1$

$= \tilde{k} \times (\tilde{i}+\tilde{j}+\tilde{k}) = -(\tilde{j}-\tilde{i}) \quad \Rightarrow \quad \frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0}$

Angle between $\lambda(-\tilde{j}+\tilde{i})$ and $(\tilde{i}-2\tilde{j}+2\tilde{k})$

$$\cos\theta = \frac{\lambda(-\tilde{j}+\tilde{i}).(\tilde{i}-2\tilde{j}+2\tilde{k})}{\lambda\sqrt{2}\times 3} = \frac{1}{\sqrt{2}}$$

$\Rightarrow \theta = \frac{\pi}{4}$ or $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Match the column :

1. **(A)** Vector parallel to line of intersection of the plane is $(\tilde{i}+\tilde{j}) \times (\tilde{j}+\tilde{k}) = \tilde{k}-\tilde{j}+\tilde{i}$

equation of line whose dr's, are (1, – 1, 1) and passing through (0, 0, 0) is

x = – y = z

(B) Similarly $(\tilde{i} \times \tilde{j}) = \tilde{k}$.

Hence dr's = (0, 0, 1)

and passing through the point (2, 3, 0)

$\therefore$ Equation of line $\frac{x-2}{0} = \frac{y-3}{0} = \frac{z}{1}$

(C) Similarly $\tilde{i} \times (\tilde{j}+\tilde{k}) = \tilde{k}-\tilde{j}$

dr's = (0, –1, 1)

Equation of line $\frac{x-2}{0} = \frac{y-2007}{-1} = \frac{z+2004}{1}$

because x = 2 & y + z = 3

so y = 2007, z = – 2004 satisfy above equation

(D) x = 2, x + y + z = 3

y + z = 1

same as part C

we get $\frac{x-2}{0} = \frac{y}{-1} = \frac{z-1}{1}$

Assertion & Reason :

1. Statement-I Equation of plane is

$(\vec{r}-\vec{a}).(\vec{b}\times\vec{c}) = 0 \qquad ....(1)$

$\vec{r} = \vec{a}+\lambda\vec{b}+\mu\vec{c}$ satisfies above equation

Hence True

Statement-II is also true & explain statement I

4. Let the coordinates of A, B, C, D be A(1, 0, 0), B(1, 1, 0), C(0, 1, 0) and D(0, 0, 0)

so that coordinates of A_1, B_1, C_1 are $A_1(1,0,1)$, $B_1(1,1,1)$, $C_1(0, 1, 1)$ & $D_1(0, 0, 1)$

The coordinates of midpoint of B_1A_1 is $P\left(1, \frac{1}{2}, 1\right)$ and that of B_1C_1 is $Q\left(\frac{1}{2}, 1, 1\right)$

Equation of the plane PBQ is 2x + 2y + z = 4

Its distance from D(0, 0, 0) is $\frac{4}{3}$

So Statement-1 is false and Statement-2 is clearly true.

7. plane P_1 is $\perp$ to $\vec{a} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 1 & -1 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 2\tilde{i}+\tilde{j}+\tilde{k}$

and plane P_2 is $\perp$ to $\vec{b} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 1 & 0 & -2 \\ 2 & -1 & -3 \end{vmatrix} = -2\tilde{i}-\tilde{j}-\tilde{k}$

$\Rightarrow \vec{a} \,||\, \vec{b} \Rightarrow P_1$ & P_2 are parallel

also L is parallel to $\vec{c} = \tilde{i}-\tilde{j}-\tilde{k}$

also $\vec{a}.\vec{c} = 0$ & $\vec{b}.\vec{c} = 0$

but it is not essential that if P_1 & P_2 are parallel to L then P_1 & P_2 must be parallel.

So Statement-II is not a correct explanation of Statement-I.

Comprehension # 1

A (2, 1, 0), B (1, 0, 1)

C (3, 0, 1) and D(0, 0, 2)

1. Equation of plane ABC

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & 1 & -1 \\ 2 & 0 & 0 \end{vmatrix} = 0 \quad \Rightarrow y + z = 1$$

2. Equation of $L = 2\tilde{k}+\lambda(\overrightarrow{AB}\times\overrightarrow{AC})$

so $L = 2\tilde{k}+\lambda(\tilde{j}+\tilde{k})$

3. Equation of plane ABC

y + z – 1 = 0

distance from (0, 0, 2) is $= \frac{2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

EXERCISE - 04[A] CONCEPTUAL SUBJECTIVE EXERCISE

4. After rotation equation of plane is new position will be

$\ell x + my + a' z = 0$ ....(1)

Let angle between (1) and $\ell x + my = 0$ is θ, then

$$\cos\theta = \frac{\ell^2 + m^2}{\sqrt{\ell^2 + m^2}\sqrt{\ell^2 + m^2 + a'^2}}$$

Solving we get

$a'^2 = (\ell^2 + m^2)\tan^2\theta$

$\Rightarrow a' = \pm\sqrt{(\ell^2 + m^2)}\tan\theta$

Equation is $\ell x + my \pm z\sqrt{(\ell^2 + m^2)}\tan\theta = 0$

5. Let point on line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ .....(1)

are $(3 + 2\lambda, 3 + \lambda, \lambda)$

Equation of line which pass through origin is

$$\frac{x-0}{3+2\lambda} = \frac{y-0}{3+\lambda} = \frac{z-0}{\lambda} \quad .....(2)$$

Angle between (1) & (2)

$$\cos\frac{\pi}{3} = \frac{(3+2\lambda)2 + (3+\lambda)1 + \lambda\times 1}{\sqrt{(3+2\lambda)^2 + (3+\lambda)^2 + \lambda^2}\sqrt{2^2 + 1^2 + 1}}$$

Solving we get

$\lambda^2 + 3\lambda + 2 = 0$

$\Rightarrow \lambda = -1, -2$

Putting the value of λ in equation (2)

$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ or $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$

7. Planes are $x - 2y + z = 1$ ....(i)

$x + 2y - 2z = 5$ ....(ii)

$2x + 2y + z = -6$ ....(iii)

Add (i) + (ii) + (iii)

$4x + 2y = 0 \Rightarrow y = -2x$ ....(iv)

From equations (iii) – (i)

$x + 4y = -7$ ....(v)

from (iv) and (v) we get

$x = 1, y = -2$

Put in (i) we get $z = -4$

So point of intersection is $(1, -2, -4)$

9. Line : $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$

Plane : $x - y + z + 2 = 0$

The vector perpendicular to required plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 1 & -1 & 2 \end{vmatrix} = 2\hat{i} + 3\hat{j} + \hat{k}$$

Now equation of plane passing through $(1, -2, 0)$ and perpendicular to $2\hat{i} + 3\hat{j} + \hat{k}$

$(x - 1)2 + (y + 2)3 + (z - 0)1 = 0$

$\Rightarrow 2x + 3y + z + 4 = 0$

EXERCISE - 04[B] BRAIN STORMING SUBJECTIVE EXERCISE

3. Angular point OABC are (0, 0, 0), (0, 0, 2), (0, 4, 0) & (6, 0, 0)

Let centre of sphere be (r, r, r)

Equation of plane passing ABC is

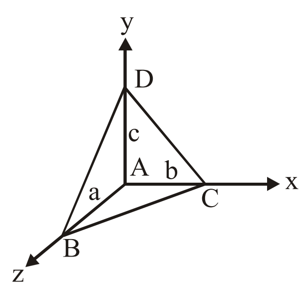
$$\frac{x}{6} + \frac{y}{4} + \frac{z}{2} = 1$$

$$r = \left|\frac{\frac{r}{6} + \frac{r}{4} + \frac{r}{2} - 1}{\sqrt{\frac{1}{6^2} + \frac{1}{4^2} + \frac{1}{2^2}}}\right|$$

$7r = \pm(11r - 12)$

$r = \frac{2}{3}$, r = 3 (not satisfied)

7.



Area of Δ ABC $\Rightarrow \frac{1}{2}ab = x$ ...(i)

Area of Δ ABC $\Rightarrow \frac{1}{2}bc = y$ ...(ii)

Area of Δ ACD $\Rightarrow \frac{1}{2}ac = z$ ....(iii)

Area of $\Delta BCD = \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}$

$= \frac{1}{2}\times 2\sqrt{x^2 + y^2 + z^2}$

$= \sqrt{x^2 + y^2 + z^2}$

5. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix} = 0$

$k^2 + 3k = 0 \Rightarrow k(k+3) = 0 \Rightarrow k = 0$ or -3

7. Let $\vec{n}_1$ and $\vec{n}_2$ be the vectors normal to the faces OAB and ABC. Then,

$$\vec{n}_1 = \overrightarrow{OA} \quad \overrightarrow{OB} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\tilde{i} - \tilde{j} - 3\tilde{k}$$

and, $\vec{n}_2 = \overrightarrow{AB} \quad \overrightarrow{AC} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \tilde{i} - 5\tilde{j} - 3\tilde{k}$

If θ is the angle between the faces OAB and ABC, then

$$\cos\theta = \frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$$\Rightarrow \cos\theta = \frac{5+5+9}{\sqrt{25+1+9}\sqrt{1+25+9}} = \frac{19}{35}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

8. $\ell_1 - am_1 = 0$ and $cm_1 - n_1 = 0 \Rightarrow \frac{\ell_1}{a} = \frac{m_1}{1} = \frac{n_1}{c}$

Also $\ell_2 - a'm_2 = 0$ and $c'm_2 - n_2 = 0$

$\Rightarrow \frac{\ell_2}{a'} = \frac{m_2}{1} = \frac{n_2}{c'}$

$\therefore \ell_1\ell_2 + m_1m_2 + n_1n_2 = aa' + cc' + 1 = 0$

9. Here, $\ell = \cos\theta$, $m = \cos\beta$, $n = \cos\theta$, $(\because \ell = n)$

Now, $\ell^2 + m^2 + n^2 = 1 \Rightarrow 2\cos^2\theta + \cos^2\beta = 1$

$\Rightarrow$ Given, $\sin^2\beta = 3\sin^2\theta \Rightarrow 2\cos^2\theta = 3\sin^2\theta$

$5\cos^2\theta = 3$, $\therefore \cos^2\theta = \frac{3}{5}$

10. Given plane are $2x + y + 2z - 8 = 0$

or $4x + 2y + 4z - 16 = 0$ ..... (i)

and $4x + 2y + 4z + 5 = 0$ ..... (ii)

Distance between two parallel planes

$$= \left|\frac{-16-5}{\sqrt{4^2+2^2+4^2}}\right| = \frac{21}{6} = \frac{7}{2}$$

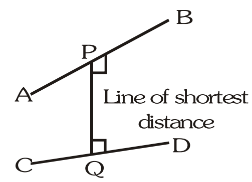
11. Let the two lines be AB and CD having equation

$\frac{x}{1} = \frac{y+a}{1} = \frac{z}{1} = \lambda$ and $\frac{x+a}{2} = \frac{y}{1} = \frac{z}{1} = \mu$

then $P \equiv (\lambda, \lambda - a, \lambda)$ and $Q = (2\mu - a, \mu, \mu)$

So according to question,

$\frac{\lambda-2\mu+a}{2} = \frac{\lambda-a-\mu}{1} = \frac{\lambda-\mu}{2}$



$\Rightarrow \mu = a$ and $\lambda = 3a$

$\therefore P \equiv (3a, 2a, 3a)$

and $Q \equiv (a, a, 0)$

Trick : Put the option and check it

12. We have, $\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s$

and $\frac{x-0}{1/2} = \frac{y-1}{1} = \frac{z-2}{-1} = t$

Since, lines are coplanar then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ 1/2 & 1 & -1 \end{vmatrix} = 0$$

On solving, $\lambda = -2$

14. Angle between line and normal to plane is

$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{1\times2-2\times1+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$, where θ is the angle between line and plane

$$\Rightarrow \sin\theta = \frac{1\times2+2\times(-1)+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}} \Rightarrow \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$$

$$\Rightarrow \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} \Rightarrow \lambda = \frac{5}{3}$$

15. The lines are $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

Since, $a_1a_2 + b_1b_2 + c_1c_2 = 6 - 24 + 18 = 0$

$\Rightarrow \theta = 90$

20. Equation of line PQ is

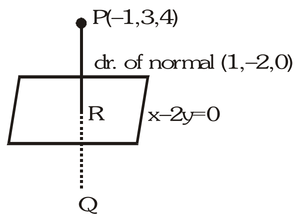
$$\frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \lambda$$

For some suitable value of λ, co-ordinates of point

$Q(\lambda - 1, 3 - 2\lambda, 4)$

R is the mid point of P and Q.

$$\therefore R \equiv \left(\frac{\lambda-2}{2}, \frac{6-2\lambda}{2}, 4\right)$$



$$R \equiv \left(\frac{\lambda}{2} - 1, 3 - \lambda, 4\right)$$

It satisfies $x - 2y = 0$

$$\Rightarrow \lambda = \frac{14}{5}$$

$$\therefore \; Q = \left(\frac{2}{5}, \frac{1}{5}, 4\right)$$

21. If direction cosines of L be ℓ, m, n then

$2\ell + 3m + n = 0$

$\ell + 3m + 2n = 0$

Solving, we get, $\frac{\ell}{3} = \frac{m}{-3} = \frac{n}{3}$

$$\therefore \; \ell : m : n = \frac{1}{\sqrt{3}} : -\frac{1}{\sqrt{3}} : \frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

22. $\ell = \cos\frac{\pi}{4}$, $m = \cos\frac{\pi}{4}$

we know that $\ell^2 + m^2 + n^2 = 1$

$$\frac{1}{2} + \frac{1}{2} + n^2 = 1 \Rightarrow n = 0$$

Hence angle with positive direction of z-axis is $\frac{\pi}{2}$

26. Line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ .... (1)

Plane $x + 3y - \alpha z + \beta = 0$ .... (2)

Point (2, 1, –2) put in (2)

$2 + 3 + 2\alpha + \beta = 0$

$\Rightarrow 2\alpha + \beta = -5$

Now $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$3 - 15 - 2\alpha = 0$

$-12 - 2\alpha = 0$

$\alpha = -6$

$-12 + \beta = -5$

$\beta = 7$

$\alpha = -6, \; \beta = 7$

27. Proj. of a vector ($\vec{r}$) on x–axis $= |\vec{r}|\,\ell$

on y–axis $= |\vec{r}|\,m$

on z–axis $= |\vec{r}|\,n$

$6 = 7\ell, \quad \Rightarrow \ell = \frac{6}{7}$ similarly $m = -\frac{3}{7}$, $n = \frac{2}{7}$

28. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)

$\alpha = 45, \; \beta = 120$

Put in equation (i)

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\gamma = \frac{1}{4}$$

$\Rightarrow \gamma = 60$

29. Mirror image of B(1, 3, 4) in plane $x-y+z = 5$

$$\frac{x-1}{1} = \frac{y-3}{-1} = \frac{z-4}{1} = -2\frac{(1-3+4-5)}{1+1+1} = 2$$

$\Rightarrow x = 3, \; y = 1, \; z = 6$

$\therefore$ mirror image of B (1, 3, 4)

is A (3, 1, 6)

statement-1 is correct

statement-2 is true but it is not the correct explanation.

30. $\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ equation of line

equation of plane $x + 2y + 3z = 4$

$$\sin\theta = \frac{1+4+3\lambda}{\sqrt{14}\sqrt{1+4+\lambda^2}}$$

$$\Rightarrow \lambda = \frac{2}{3}$$

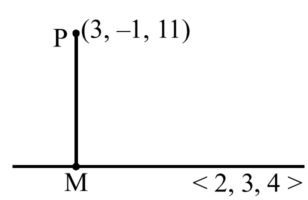
31. $1(1-1) + 2(0 - 6) + 3(7 - 3)$

$= 0 - 12 + 12 = 0$

mid point AB (1, 3, 5)

lies on $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

32.



$M(2r, 3r + 2, 4r + 3)$

Dr's of PM $\langle 2r - 3, 3r + 3, 4r - 8 \rangle$

$2(2r - 3) + 3(3r + 3) + 4(4r - 8) = 0$

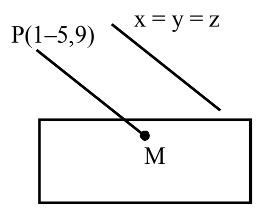
$29r - 29 = 0$

$r = 1$

$M(2, 5, 7)$

Distance PM $= \sqrt{1+36+16} = \sqrt{53}$

33.



eq^n. of a line || to x = y = z and passing through (1, –5, 9) is

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=r$$

Let is meets plane at M(r+1, r–5, r+9)

Put in equation of plane

$x - y + z = 5$

$r + 1 - r + 5 + r + 9 = 5$

$r = -10$

Hence M (–9, –15, –1)

Distance PM $=\sqrt{100+100+100}=10\sqrt{3}$

34. Equation of plane parallel to

$x - 2y + 2z - 5 = 0$

is $x - 2y + 2z = k$

or $\frac{x}{3}-\frac{2}{3}y+\frac{2}{3}z=\frac{K}{3}$

$$\left|\frac{K}{3}\right| = 1$$

$\Rightarrow K = \pm 3$

$\therefore$ Equation of required plane is

$x - 2y + 2z \pm 3 = 0$

35. $$\begin{vmatrix} 3-1 & K+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & K+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2K - 9 = 0$

$\Rightarrow K = \frac{9}{2}$

36. $4x + 2y + 4z + 5 = 0$

$4x + 2y + 4z - 16 = 0$

$$\Rightarrow \quad d=\left|\frac{21}{\sqrt{36}}\right|=\frac{7}{2}$$

37. $\Rightarrow (\vec{a}-\vec{b}).(\vec{c}\times\vec{d})=0$

$$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow (1 + 2k) + (1 + k^2) - (2 - k) = 0$

$\Rightarrow k^2 + 3k = 0 \begin{cases} 0 \\ -3 \end{cases}$

3. Let the equation of the plane ABCD be $ax + by + cz + d = 0$, the point A" be (α, β, γ) and the height of the parallelopiped ABCDA'B'C'D' be h.

$$\Rightarrow \quad \frac{|a\alpha + b\beta + c\gamma + d|}{\sqrt{a^2+b^2+c^2}} = 90\%.h$$

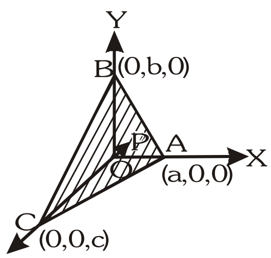
$$\Rightarrow \quad a\alpha + b\beta + c\gamma + d = \pm 0.9h\sqrt{a^2+b^2+c^2}$$

$\therefore$ locus is ax, + by + cz + d = $\pm 0.9h\sqrt{a^2+b^2+c^2}$

$\therefore$ locus of A" is a plane parallel to the plane ABCD.

6. As $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate axes at A(a, 0, 0), B(0, b, 0), C(0, 0, c) and its distance from origin = 1

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = 1$$



or $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = 1$ ... (1)

where P is centroid of Δ

$$\therefore \quad P(x, y, z) = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$\Rightarrow \quad x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3} \quad ... (2)$$

Thus, from (1) and (2)

$$\frac{1}{9x^2}+\frac{1}{9y^2}+\frac{1}{9z^2} = 1$$

or $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} = 9 = K$

$\therefore$ K = 9

7. Equation of plane containing the line,

$2x - y + z - 3 = 0$ and $3x + y + z = 5$ is

$(2x - y + z - 3) + \lambda(3x + y + z - 5) = 0$

$\Rightarrow (2+3\lambda)x + (\lambda - 1)y + (\lambda + 1)z - 3 - 5\lambda = 0$

Since distance of plane from (2, 1, –1) to above plane is $1/\sqrt{6}$

$$\therefore \quad \left|\frac{6\lambda+4+\lambda-1-\lambda-1-3-5\lambda}{\sqrt{(3\lambda+2)^2+(\lambda-1)^2+(\lambda+1)^2}}\right| = \frac{1}{\sqrt{6}}$$

$\Rightarrow \quad 6(\lambda - 1)^2 = 11\lambda^2 + 12\lambda + 6$

$\Rightarrow \quad \lambda = 0, -\frac{24}{5}$

$\therefore$ Equation of planes are,

$2x-y+z-3 = 0$ and $62x+29y+19z-105 = 0$

9. (A) Solving the two equations, say i.e.,

$x + y = |a|$ and $ax - y = 1$, we get

$x = \frac{|a|+1}{a+1} > 0$ and $y = \frac{|a|-1}{a+1} > 0$

when $a + 1 > 0$; we get $a > 1$

$\therefore \quad a_0 = 1$

(B) We have, $\vec{a} = \alpha\tilde{i} + \beta\tilde{j} + \gamma\tilde{k}$

$\Rightarrow \vec{a}.\tilde{k} = \gamma$

Now, $\tilde{k} \times (\tilde{k} \times \vec{a}) = (\tilde{k}.\vec{a})\tilde{k} - (\tilde{k}.\tilde{k})\vec{a}$

$= \gamma\tilde{k} - (\alpha\tilde{i} + \beta\tilde{j} + \gamma\tilde{k})$

$\Rightarrow \alpha\tilde{i} + \beta\tilde{j} = 0$

$\Rightarrow \alpha = \beta = 0$

Also $\alpha + \beta + \gamma = 2$

$\Rightarrow \quad \gamma = 2.$

(C) $\left|\int_0^1 \left(1-y^2\right)dy\right| + \left|\int_0^1 \left(y^2-1\right)dy\right|$

$= 2\int_0^1 (1-y^2)dy = \frac{4}{3}$

Also $\left|\int_0^1 \sqrt{1-x}\,dx\right| + \left|\int_{-1}^0 \sqrt{1+x}\,dx\right|$

$= 2\int_0^1 \sqrt{1-x}\,dx = \frac{4}{3}$

(D) sin A sin B sin C + cos A cos B

$\le$ sinA sinB + cosA cosB = cos(A – B)

$\Rightarrow \quad \cos(A - B) \ge 1$

$\Rightarrow \quad \cos(A - B) = 1$

$\Rightarrow \quad \sin C = 1.$

10. (A) $\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = t$.

$\Rightarrow \sum_{i=1}^{\infty} \tan^{-1}\left(\frac{2}{4i^2 - 1 + 1}\right)$

$= \sum_{i=1}^{\infty} \tan^{-1}\left\{\frac{(2i+1)-(2i-1)}{1+(2i-1)(2i+1)}\right\}$

$= (\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}5 - \tan^{-1}3) + \dots + \{(\tan^{-1}(2n+1) - \tan^{-1}(2n-1))$

$\therefore \quad t = \lim_{n\to\infty} (\tan^{-1}(2n+1) - \tan^{-1} 1)$

$= \lim_{n\to\infty} \tan^{-1}\left(\frac{2n}{1+2n+1}\right) = \frac{\pi}{4}$

$\therefore \quad \tan t = 1$.

(B) We have, $\cos\theta_1 = \dfrac{1-\tan^2\frac{\theta_1}{2}}{1+\tan^2\frac{\theta_1}{2}} = \dfrac{a}{b+c}$

$\Rightarrow \quad \tan^2\left(\frac{\theta_1}{2}\right) = \frac{b+c-a}{b+c+a}$

Also, $\cos\theta_3 = \dfrac{1-\tan^2\frac{\theta_3}{2}}{1+\tan^2\frac{\theta_3}{2}} = \dfrac{c}{a+b}$

$\Rightarrow \quad \tan^2\frac{\theta_3}{2} = \frac{a+b-c}{a+b+c}$

$\Rightarrow \quad \tan^2\frac{\theta_1}{2} + \tan^2\frac{\theta_3}{2}$

$= \frac{2b}{a+b+c} = \frac{2b}{3b} = \frac{2}{3}$

{as, a, b, c are in AP $\Rightarrow 2b = a + c$}

(C) Line through (0, 1, 0) and perpendicular to plane $x + 2y + 2z = 0$ is given by

$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-0}{2} = r$

$\therefore$ P(r, 2r + 1, 2r) be the foot of perpendicular on the straight line then

$r \cdot 1 + (2r + 1) \cdot 2 + (2r) \cdot 2 = 0$

$\Rightarrow \quad r = -\frac{2}{9}$

$\therefore \quad P\left(-\frac{2}{9}, \frac{5}{9}, -\frac{4}{9}\right)$

$\therefore$ Required perpendicular distance

$= \sqrt{\frac{4+25+16}{81}} = \frac{\sqrt{5}}{3}$ unit.

11. Let $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \frac{-1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$

(A) If $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Rightarrow \Delta = 0$ and $a = b = c \neq 0$

$\Rightarrow$ the equation represent identical planes.

(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

$\Rightarrow \Delta = 0$

Since all the three planes pass through (1,1,1)

So equation of the line of intersection of these plane will be $\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0}$

(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

$\Rightarrow \Delta \neq 0$

$\Rightarrow$ the equations represent planes meeting at only one point i.e. (0,0,0)

(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow a = b = c = 0$

$\Rightarrow$ the equations represent whole of the three dimensional space.

13. Dr's of

$L_1 = 0, -4, -4$

$L_2 = 0, -2, -2$

$L_3 = 0, 2, 2$

So all the three lines are parallel

Hence St.-I is false

Now $\Delta = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = 0$

so there will be no solution.

Hence St.-II is true.

Paragraph for Question 14 to 16

14. $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$

$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

a vector perpendicular to L_1 & L_2 will be

$= \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -i - 7j + 5k$

Hence unit vector $= \frac{-i-7j+5k}{5\sqrt{3}}$

15. Shortest distance

$= (3i - 4k).\frac{(-i-7j+5k)}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$

16. Eq. of plane $-(x+1) - 7(y+2) + 5(z+1) = 0$

$$x + 7y - 5z + 10 = 0$$

distance from (1, 1, 1) = $\frac{1+7-5+10}{5\sqrt{3}} = \frac{13}{5\sqrt{3}}$

17. Let DC's be $(\cos\alpha, \cos\alpha, \cos\alpha)$

$3c\cos^2\alpha = 1$

$\cos\alpha = \frac{1}{\sqrt{3}}$

Line PQ is $\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1/\sqrt{3}} = \lambda$

$Q\left(\frac{\lambda}{\sqrt{3}}+2, \frac{\lambda}{\sqrt{3}}-1, \frac{\lambda}{\sqrt{3}}+2\right)$

Putting in plane

$\frac{2\lambda}{\sqrt{3}} + 4 + \frac{\lambda}{\sqrt{3}} - 1 + \frac{\lambda}{\sqrt{3}} + 2 = 9$

$\frac{4\lambda}{\sqrt{3}} = 4$

$\lambda = \sqrt{3}$

Q = (3, 0, 3)

$(PQ)^2 = 1+1+1$

$PQ = \sqrt{3}$

18. Let Q be $(1 - 3\mu, \mu - 1, 5\mu + 2)$

$\Rightarrow \overrightarrow{PQ} = (-3\mu - 2)\tilde{i} + (\mu - 3)\tilde{j} + (5\mu - 4)\tilde{k}$

$\Rightarrow \overrightarrow{PQ}\cdot\tilde{n} = 0$ (where $\tilde{n}$ is $\perp^{er}$ to plane)

$\Rightarrow (-3\mu - 2)\,1 + (\mu - 3)\,.\,(-4) + (5\mu - 4)\,3 = 0$

$\Rightarrow \mu = \frac{1}{4}$.

19. (A) $f(x) = xe^{\sin x} - \cos x$

$f(0) = -1$

$f(\pi/2) = \frac{\pi}{2}e$

$f'(x) = xe^{\sin x}\cos x + e^{\sin x} > 0$

(B) $\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow k(k-4) - 4c + 8 - 2k = 0$

$\Rightarrow k^2 - 4k + 8 - 2k = 0$

$\Rightarrow k^2 - 6k + 8 = 0$

$\Rightarrow k = 2, 4$

(C) $|x-1| + |x-2| + |x+1| + |x+2| = 4k$

–3 –2 –1 0 1 2 3

$4k = 8, 12, 16, 20$

$\therefore k = 2, 3, 4, 5.$

{modulus denotes the distance of x from –2, –1, 1, 2

(D) $\frac{dy}{y+1} = dx$

$\ln(y+1) = ke^x$

$y + 1 = ke^x$

$y + 1 = 2 = k$

$y + 1 = 2e^x$

$y = (2e^x - 1)$

$y(\ln 2) = 3.$

36 Normal vector to the plane containing the lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

$\tilde{n} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\tilde{i} - \tilde{j} - 10\tilde{k}$

Let direction ratios of required plane be a, b, c.
Now $8a - b - 10c = 0$
and $2a + 3b + 4c = 0$

($\because$ plane contains the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$)

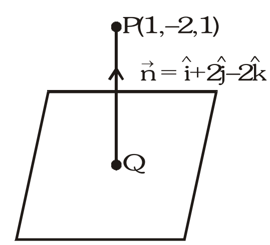
$\Rightarrow \frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$

$\Rightarrow$ equation of plane is $x - 2y + z = d$

$\because$ plane contains the line, which passes through origin, hence origin lies on a plane.

$\Rightarrow$ equation of required plane is $x - 2y + z = 0$.

21. $\because \left|\frac{1-4-2-\alpha}{3}\right| = 5$



$\Rightarrow \alpha = 10, -20$

$\Rightarrow \alpha = 10 \because \alpha > 0$

Now, let $Q(\alpha,\beta,\gamma)$ be the foot of perpendicular from P to the plane $x + 2y - 2z = 10$

Equation of line PQ is

$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = r$ (Let)

$\Rightarrow \alpha = r + 1,\ \beta = 2r - 2$ and $\gamma = -2r + 1$

$\because$ Q lies in the plane

$\therefore (r+1) + 2(2r-2) - 2(-2r+1) = 10$

$\Rightarrow r = \frac{5}{3}$

foot of the perpendicular is $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$

22. Plane containing the line

Direction ratio's of normal to the plane :

$$\begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\tilde{i}+2\tilde{j}-\tilde{k}$$

Hence equation of plane $1(x - 1) - 2(y - 2) + 1(z - 3) = 0$

i.e. $x - 2y + z = 0$

As given plane must be parallel $\Rightarrow$ A = 1

& distance between the planes

$$\left|\frac{d-0}{\sqrt{1^2+2^2+1^2}}\right| = \sqrt{6}$$

$|d| = 6$

23. (A) $P(\lambda+2, -2\lambda+1, \lambda-1)$

$$Q\left(2k+\frac{8}{3}, -k-3, k+1\right)$$

$3\lambda+6 = a(6k+8)$ ....(i)

$-2\lambda + 1 = a(-k - 3)$ ....(ii)

$2\lambda - 2 = 2a(k + 1)$..........(iii)

(ii) + (iii) $\Rightarrow$ $-1 = ak - a$

$k = \frac{a-1}{a}$ ....(iv)

Put the value of k in equation (iii)

$\Rightarrow$ $\lambda = 2a$ ....(v)

Put the values of λ & k in equation (i)

$6a+6 = a\left(\frac{6a-6}{a}+8\right) \Rightarrow 6 = 6a - 6 + 8a$

$\Rightarrow$ $a = \frac{3}{2}$

Put the value of a in equation (iv) & (v)

$k = \frac{\frac{3}{2}-1}{\frac{3}{2}} = \frac{1}{3}$ & $\lambda = 3$

$P(5, -5, 2)$ & $Q\left(\frac{10}{3},\frac{-10}{3},\frac{4}{3}\right)$

$$d = \sqrt{\left(5-\frac{10}{3}\right)^2+\left(5-\frac{10}{3}\right)^2+\left(\frac{2}{3}\right)^2}$$

$$= \sqrt{\frac{25}{9}+\frac{25}{9}+\frac{4}{9}}$$

$\Rightarrow$ $d = \sqrt{6}$ $\Rightarrow$ $d^2 = 6$

(B) $\tan^{-1}(x + 3) - \tan^{-1}(x - 3) = \tan^{-1}\left(\frac{3}{4}\right)$

$$\tan^{-1}\left(\frac{(x+3)-(x-3)}{1+(x^2-9)}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

$\Rightarrow$ $1 + x^2 - 9 = 8$ $\Rightarrow$ $x^2 = 16$

$\Rightarrow$ $x = \pm 4$

(C) $\mu b^2 + 4\,\vec{b}.\vec{c} = 0$

$b^2 - \vec{a}.\vec{c} + \vec{b}.\vec{c} = 0$

$b^2 - \left(\mu\vec{b}+4\vec{c}\right).\vec{c} + \vec{b}.\vec{c}$

$= b^2 + \vec{b}.\vec{c}\left(1-\mu\right) - 4c^2 = 0$

$b^2 - \frac{\mu}{4}b^2\left(1-\mu\right) = 4c^2$

$b^2\left(4-\mu+\mu^2\right) = 16c^2$ ....(i)

$4b^2 + 8\vec{b}.\vec{c} + 4c^2 = b^2 + a^2$

$3b^2 - 2\mu b^2 + 4c^2 = \left(\mu\vec{b}+4\vec{c}\right)^2$

$3b^2 - 2\mu b^2 + 4c^2 = \mu^2 b^2 + 8\mu\vec{b}.\vec{c} + 16c^2$

$b^2\left(3-2\mu-\mu^2\right) = 12c^2 - 2\mu^2 \times b^2$

$b^2(3 - 2\mu + \mu^2) = 12c^2$ ....(ii)

$\frac{4-\mu+\mu^2}{3-2\mu+\mu^2} = \frac{4}{3}$

$12 - 3\mu + 3\mu^2 = 12 - 8\mu + 4\mu^2$

$\mu^2 - 5\mu = 0$

$\mu = 0, 5$

(D) $I = \frac{2}{\pi}\int_{-\pi}^{\pi}\frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}dx$

$I = \frac{4}{\pi}\int_{0}^{\pi}\frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}dx$ .... (i)

$I = \frac{4}{\pi}\int_{0}^{\pi}\frac{\cos\frac{9x}{2}}{\cos\frac{x}{2}}dx$ .... (ii)

(i) + (ii)

$$I = \frac{2}{\pi}\int_0^{\pi} \frac{\sin 5x}{\sin\frac{x}{2}\cos\frac{\pi}{2}}dx = \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x}{\sin x}dx$$

$f(x) = f(\pi - x)$

$$I = \frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 5x}{\sin x}dx \qquad \text{....... (i)}$$

$$I = \frac{8}{\pi}\int_0^{\pi/2}\frac{\cos 5x}{\cos x}dx \qquad \text{....... (ii)}$$

(i) + (ii)

$$I = \frac{4}{\pi}\int_0^{\pi/2}\frac{\sin 6x}{\sin x \cos x}dx$$

$$I = \frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 6x}{\sin 2x}dx = \frac{8}{\pi}\int_0^{\pi/2}(3 - 4\sin^2 2x)dx$$

$$= \frac{8}{\pi}\int_0^{\pi/2} 3 - 2(1 - \cos 2x)dx$$

$$= \frac{8}{\pi}\int_0^{\pi/2}(1 + 2\cos 2x)dx = \frac{8}{\pi}\times\frac{\pi}{2} = 4$$

24. **(a)** Line QR : $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1} = \lambda$

Any point on line QR :

$(\lambda + 2,\ 4\lambda + 3,\ \lambda + 5)$

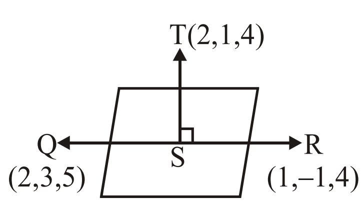
$\therefore$ Point of intersection with plane :

$5\lambda + 10 - 16\lambda - 12 - \lambda - 5 = 1$

$\Rightarrow \quad \lambda = -\frac{2}{3}$

$\therefore \quad P\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Also



$\because$ TQ = TR = $\sqrt{5}$

$\Rightarrow$ S is the mid-point of QR

$\Rightarrow \quad S\left(\frac{3}{2}, 1, \frac{9}{2}\right) \quad \Rightarrow \quad PS = \frac{1}{\sqrt{2}}$ units

(b) Let required plane be $(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\because$ plane is at a distance $\frac{2}{\sqrt{3}}$ from the point (3,1,–1).

$$\Rightarrow \left|\frac{(3+2-3-2)+\lambda(3-1-1-3)}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \lambda^2 = \frac{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}{3}$$

$\Rightarrow 3\lambda^2 = 3\lambda^2 + 2\lambda - 4\lambda + 6\lambda + 14$

$\Rightarrow \lambda = -\frac{7}{2}$

$\therefore$ required plane is $(x + 2y + 3z - 2)$

$$+\left(-\frac{7}{2}\right)(x - y + z - 3) = 0$$

$\Rightarrow 5x - 11y + z = 17$

(c) (1, –1, 0); (–1, –1, 0)

For coplanarity of lines

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \qquad \Rightarrow \qquad 2(k^2 - 4) = 0$$

$\Rightarrow \quad k = \pm 2$

for k = 2

Normal vector $\vec{n} = \tilde{j} - \tilde{k}$

$\therefore$ Required plane : $y - z = \lambda$

$\because$ Passes through (1, –1, 0)

$\Rightarrow \quad \lambda = -1$

$\therefore \quad y - z = -1$

for k = –2

$\vec{n} = \tilde{j} + \tilde{k}$

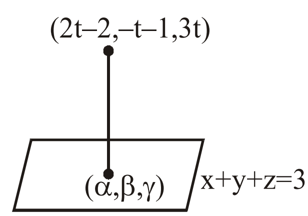
$\therefore$ Required plane : $y + z = \lambda$

$\because$ Passes through (1, –1, 0)

$\Rightarrow \quad \lambda = -1$

$\therefore \quad y + z = -1$

25. $\frac{\alpha - 2t + 2}{1} = \frac{\beta + t + 1}{1} = \frac{\gamma - 3t}{1} = k$



$\alpha = k + 2t - 2$

$\beta = k - t - 1$

$\gamma = k + 3t$

$$\alpha + \beta + \gamma = 3$$

$$k = \frac{6-4t}{3}$$

$$\alpha = \frac{6-4t}{3} + 2t - 2 = \frac{2t}{3}$$

$$\beta = \frac{6-4t}{3} - t - 1 = \frac{3-7t}{3}$$

$$\gamma = \frac{6-4t}{3} + 3t = \frac{5t+6}{3}$$

$$\Rightarrow \frac{3\alpha}{2} = \frac{3\beta-3}{-7} = \frac{3\gamma-6}{5}$$

$$\Rightarrow \frac{x}{2} = \frac{y-3}{-7} = \frac{z-2}{5}$$

26. $\ell_1 : \vec{r} = (3, -1, 4) + (1, 2, 2)t$

$\ell_2 : \vec{r} = (3, 3, 2) + (2, 2, 1)s$

vector perpendicular to ℓ_1 and ℓ_2 :

$$\begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\tilde{i} + 3\tilde{j} - 2\tilde{k}$$

$\therefore$ Equation of line ℓ : $\vec{r} = 0 + (-2, 3, -2)\lambda$

Point of intersection of ℓ_1 and ℓ :

$3 + t = -2\lambda$

$-1 + 2t = 3\lambda.$

$4 + 2t = -2\lambda.$

On solving we get $\lambda = -1$, $t = -1$

$\therefore$ Point of intersection of ℓ_1 & ℓ : P(2, –3, 2)

A point on ℓ_2 at distance of $\sqrt{17}$ from P :

$\Rightarrow$ $(1 + 2s)^2 + (6 + 2s)^2 + s^2 = 17$

$\Rightarrow$ $s = -\frac{10}{9}; s = -2$

for above s, point will be (B), (D)

27. $L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$

$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$

for lines to be coplanar

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow$ $(5 - \alpha)((3 - \alpha)(2-\alpha) - 2) = 0$

$\Rightarrow$ $(5 - \alpha)(\alpha^2 - 5\alpha + 4) = 0$

$\Rightarrow$ $\alpha = 1, 4, 5$

28. For point of intersection of L_1 and L_2

$$\begin{cases} 2\lambda + 1 = \mu + 4 \\ -\lambda = \mu - 3 \\ \lambda - 3 = 2\mu - 3 \end{cases} \Rightarrow \mu = 1$$

$\Rightarrow$ point of intersction is (5, –2, –1)

Now, vector normal to the plane is

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = -16(\tilde{i} - 3\tilde{j} - 2\tilde{k})$$

Let equation of required plane be

$x - 3y - 2z = \alpha$

$\because$ it passes through (5, –2, –1)

$\therefore$ $\alpha = 13$

$\Rightarrow$ equation of plane is $x - 3y - 2z = 13$