TANGENT & NORMAL

EXERCISE - 01 CHECK YOUR GRASP

1. Let any point on the curve is $\left(\frac{c}{t}, ct^2\right)$

$$y = \frac{c^3}{x^2} \Rightarrow \frac{dy}{dx} = -\frac{2c^3}{x^3} = -\frac{2c^3}{c^3/t^3}$$

$$\frac{dy}{dx} = -2t^3$$

Equation of tangent is

$$y - ct^2 = -2t^3\left(x - \frac{c}{t}\right)$$

For x intercept

$$0 - ct^2 = -2t^3\left(a - \frac{c}{t}\right) \Rightarrow \frac{c}{2t} = a - \frac{c}{t}$$

$$a = \frac{3}{2}\frac{c}{t}$$

For y intercept

$$b - ct^2 = -2t^3\left(0 - \frac{c}{t}\right)$$

$$\Rightarrow b - ct^2 = 2t^2c \quad \Rightarrow b = 3ct^2$$

$$\text{Now } a^2b = \frac{9}{4}\cdot\frac{c^2}{t^2} \quad 3ct^2 = \frac{27c^3}{4}$$

8. Let the line has equation $y = c$

Now given curve is $y = \sqrt{x}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

Now $c = \sqrt{x}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2c}$$

$$\text{since} \tan\frac{\pi}{4} = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

$$\Rightarrow 1 = \left|\frac{\frac{1}{2c} - 0}{1 + 0}\right| \quad \Rightarrow \frac{1}{2c} = \pm 1 \Rightarrow c = \pm\frac{1}{2}$$

But c is positive $\Rightarrow y = \frac{1}{2}$

10. $2y = x^3 \Rightarrow \frac{dy}{dx} = \frac{3}{2}x^2 = 0$

$$y^2 = 32x \Rightarrow 2y\frac{dy}{dx} = 32 \quad \Rightarrow \frac{dy}{dx} \to \infty$$

Angle between the curves is $\pi/2$

12.

$$\sqrt{\frac{y\frac{dy}{dx}}{\frac{y}{\frac{dy}{dx}}}} = \sqrt{\left(\frac{dy}{dx}\right)^2} = \left|\frac{dy}{dx}\right|$$

14. Length of subnormal $= y\frac{dy}{dx}$

$$y^2 = 8ax \quad \Rightarrow 2y\frac{dy}{dx} = 8a$$

$$\therefore \quad y\frac{dy}{dx} = 4a$$

17. $\left|\frac{dx}{dt}\right| < \left|\frac{dy}{dt}\right|$

$$\left|\frac{dy}{dx}\right| > 1$$

$$3y^2\frac{dy}{dx} = 27 \Rightarrow \frac{dy}{dx} = \frac{9}{y^2}$$

$$\frac{9}{y^2} > 1 \quad \Rightarrow \quad y^2 < 9$$

$$\Rightarrow \quad -3 < y < 3 \quad \Rightarrow \quad -27 < y^3 < 27$$

$$\Rightarrow \quad -27 < 27x < 27 \quad \Rightarrow \quad -1 < x < 1$$

19. (a) $2y\frac{dy}{dx} = 4a \Rightarrow \left(\frac{dy}{dx}\right)_1 = \frac{2a}{y_1} = \frac{2a}{e^{-x/2a}} = m_1$

For II[nd] curve $\left(\frac{dy}{dx}\right)_2 = \frac{-1}{2a}e^{\frac{-x}{2a}} = m_2$

$m_1 m_2 = -1$

(b) $2y\left(\frac{dy}{dx}\right)_1 = 4a \quad ; \quad 2x = 4a\left(\frac{dy}{dx}\right)_2$

$m_1 = \frac{2a}{y_1} \qquad m_2 = \frac{x_1}{2a}$

$y_1^2 = 4ax_1$ ....(i) $\qquad x_1^2 = 4ay_1$ ......(ii)

$m_1m_2 \neq -1$

(c) $y = \frac{a^2}{x}$; $\qquad x^2 - y^2 = b^2$

$m_1 = -\frac{a^2}{x_1^2}$; $2x_1 - 2y_1m_2 = 0 \Rightarrow m_2 = \frac{x_1}{y_1}$

$m_1m_2 = \frac{-a^2}{x_1y_1} = \frac{-a^2}{a^2} = -1$

(d) $m_1 = \frac{dy}{dx} = a$; $\quad 2x + 2ym_2 = 0$

$m_2 = -\frac{x}{y}$

$m_1m_2 = -\frac{ax}{y} = -\frac{ax}{ax} = -1$

22. $\frac{dx}{dt} = \frac{2\left(-\operatorname{cosec}^2 t\right)}{\cot t}$

at $t = \frac{\pi}{4}, \frac{dx}{dt} = -4$

$$\frac{dy}{dt} = \sec^2 t - \operatorname{cosec}^2 t$$

at $t = \frac{\pi}{4}$ $\frac{dy}{dt} = 0$

$\frac{dy}{dx} = 0$ for tangent & hence it is parallel to x–axis

& its normal is parallel to y axis

23. $f'(x) = \frac{1}{3x^{2/3}}$

$f'(0) \to \infty$ tangent is vertical at x = 0

Equation of tangent at (0, 0) is x = 0

Equation of normal is y = 0

$f(x) = f^{-1}(x)$

$x^{\frac{1}{3}} = x^3 \Rightarrow x^9 = x$

$\Rightarrow x = 0 ; 1 ; -1$

EXERCISE - 02 BRAIN TEASERS

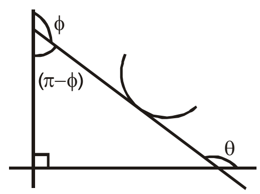
1. $\frac{dy}{dx} = K^2 e^{kx}$

$\left.\frac{dy}{dx}\right|_{x=0} = K^2 = \tan\theta$

(where θ is angle made by x-axis)

Let φ be the angle made by y-axis

$\tan\theta = \tan\left(\frac{3\pi}{2} - \phi\right) = \cot\phi$



$\cot\phi = K^2$

$\phi = \cot^{-1}(K^2)$

$\Rightarrow \phi = \sin^{-1}\left(\frac{1}{\sqrt{1+K^4}}\right)$

3. $y = \frac{(a+x)^2}{x} \Rightarrow y = \frac{a^2}{x} + 2a + x$

$\frac{dy}{dx} = \frac{-a^2}{x^2} + 1 = -1$ (for equal intercepts)

$x^2 = \frac{a^2}{2} \Rightarrow x = \pm\frac{a}{\sqrt{2}}$

8. $y = x^n \Rightarrow y' = nx^{n-1}$

equation of normal $(y - a^n) = \frac{-1}{na^{n-1}}(x - a)$

$\therefore b = \frac{a^{2-n}}{n} + a^n$

$\lim_{a\to 0} b = \frac{1}{2} \Rightarrow n = 2$

10. Equation of tangent is $y - 2 = m\left(x - \frac{1}{2}\right)$

$y = mx + 2 - \frac{m}{2}$

Put it in the parabolas $mx + 2 - \frac{m}{2} = -\frac{x^2}{2} + 2$

$\frac{x^2}{2} + mx - \frac{m}{2} = 0$

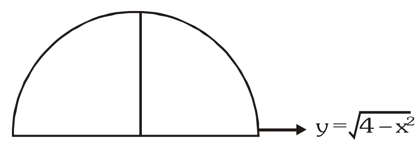
since D = 0 $\Rightarrow m^2 + m = 0$

m = 0, –1

Two tangents are there (i) y = 2

(ii) $y = -x + 2 + \frac{1}{2}$

$\Rightarrow y = -x + \frac{5}{2}$



The line y = 2 is tangent but $y = -x + \frac{5}{2}$ is secant for the curve

12. $(f'(x))^2 = f(x)f''(x)$

$\Rightarrow (y')^2 = yy''$

$\Rightarrow \int \frac{y'}{y}dx = \int \frac{y''}{y}dx \Rightarrow \ell ny = \ell ny' + c_1$

$f(0) = 1,\ f'(0) = 1 \Rightarrow c_1 = 0$

$\therefore y = y'$

$\Rightarrow \int \frac{y'}{y}dx = \int 1\,dx \Rightarrow \ell ny = x + c_2$

$f(0) = 1 \Rightarrow c_2 = 0$

$\therefore y = e^x$

$y'' > 0 \ \forall x \in R$

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Match the Column :

2. (A) $4y\frac{dy}{dx} = 2ax \Rightarrow -4\frac{dy}{dx} = 2a$

$\Rightarrow \frac{dy}{dx} = \frac{-a}{2} = -1 \Rightarrow a = 2$

$2y^2 = ax^2 + b$

$2 = a + b$

$b = 0$

$a - b = 2 - 0 = 2$

(B) Slope of normal = –1

Slope of tangent = 1 = $\frac{dy}{dx}$

$18y\frac{dy}{dx} = 3x^2$

$18b = 3a^2$

$b = \frac{a^2}{6}$ .....(i)

$9b^2 = a^3$ .....(ii)

9. $\frac{a^4}{36} = a^3 \Rightarrow a = 4$; $b = \frac{16}{6} = \frac{8}{3}$

$a - b = 4 - \frac{8}{3} = \frac{4}{3}$

(C) (1, 2) satisfies $y = ax^2 + bx + \frac{7}{2}$

$\Rightarrow \quad 2 = a + b + \frac{7}{2} \quad \Rightarrow \quad a + b = \frac{-3}{2}$

$\frac{dy}{dx} = 2ax + b = 2a + b$

for II[nd] curve $\frac{dy}{dx} = 2x + 6 = 2$

Slope of normal $= -\frac{1}{2}$

$2a + b = -\frac{1}{2}$

Solve for a & b

(D) Put, (1, 1) $\quad 1 + a + b = 0$ ......(i)

$\frac{dy}{dx} = 2$

$y + xy' + a + by' = 0$

$1 + 2 + a + 2b = 0$

$a + 2b = -3$ ......(ii)

get the values of a & b

Assertion and Reason :

3. $\frac{dy}{dx} = 7x^6 + 24x^2 + 2$

which is always positive

Comprehension # 1 :

$f(x) = x^2f(1) - xf'(2) + f''(3)$

$f(0) = 2 \Rightarrow f''(3) = 2$

$f(x) = x^2f(1) - xf'(2) + 2$

$f'(x) = 2xf(1) - f'(2)$

$f'(2) = 4f(1) - f'(2)$ ......(i)

$f''(x) = 2f(1)$

$f''(3) = 2f(1)$

$2 = 2f(1) \quad \Rightarrow \quad f(1) = 1$

$f'(2) = 4(1) - f'(2)$ (from (i))

$f'(2) = 2$

$f(x) = x^2 - 2x + 2$

1. $f'(x) = 2x - 2$

$\Rightarrow \quad f'(1) = 0$

2. $f'(x) = 2x - 2 \quad \Rightarrow \quad f'(3) = 4$

equation of tangent at (3, 5) is

$y - 5 = 4(x - 3)$

$y = 4x - 7$

3. $2e^{2x} = x^2 - 2x + 2$

interseting at (0, 2)

$\left(\frac{dy}{dx}\right)_1 = -2 \quad ; \quad \left(\frac{dy}{dx}\right)_2 = 4$

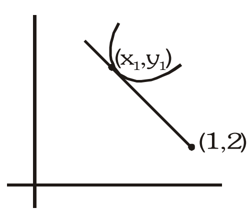
angle of intersection $= \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$\tan\theta = \left|-\frac{6}{7}\right| \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{6}{7}\right)$

EXERCISE - 04 [A] — *CONCEPTUAL SUBJECTIVE EXERCISE*

1. $2yy' - 6x^2 - 4y' = 0 \Rightarrow y' = \frac{3x_1^2}{(y_1 - 2)}$

Also slope $= \frac{y_1 - 2}{(x_1 - 1)}$



Equating both terms $\frac{3x_1^2}{(y_1 - 2)} = \frac{(y_1 - 2)}{(x_1 - 1)}$

$\Rightarrow \quad 3x_1^2(x_1 - 1) = (y_1 - 2)^2$

$\Rightarrow \quad 3x_1^3 - 3x_1^2 = y_1^2 - 4y_1 + 4$

$\Rightarrow \quad 3x_1^3 - 3x_1^2 = (2x_1^3 - 8) + 4$

$\Rightarrow \quad x_1^3 - 3x_1^2 + 4 = 0$

$\Rightarrow \quad (x_1 + 1)(x_1^2 - 4x_1 + 4) = 0$

$\Rightarrow \quad (x_1 + 1)(x_1 - 2)^2 = 0$

get the equation of tangent at $x_1 = -1, \quad x_1 = 2$

5. Slope $= -\frac{1}{2}$

$\frac{dy}{dx} = -\sin(x + y)(1 + y')$

$\Rightarrow \quad -\frac{1}{2} = -\sin(x + y)\left(1 - \frac{1}{2}\right)$

$\Rightarrow \quad \sin(x + y) = 1$

$\Rightarrow \quad \cos(x + y) = 0 \quad \Rightarrow \quad y = 0$

$\cos x = 0$

$\therefore \quad x = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$

$\sin x = 1$ is possible for $x = \frac{\pi}{2}$ or $-\frac{3\pi}{2}$

Equation are : $\quad y - 0 = -\frac{1}{2}\left(x - \frac{\pi}{2}\right)$

and $\quad y - 0 = -\frac{1}{2}\left(x + \frac{3\pi}{2}\right)$

8. At $t = 0$ the point is origin

$$\frac{dx}{dt} = \lim_{t\to 0}\frac{2t + t^2\sin 1/t - 0}{t} = 2$$

$$\frac{dy}{dt} = \lim_{t\to 0}\frac{\frac{1}{t}\sin t^2}{t} = 1$$

$$\frac{dy}{dx} = \frac{1}{2}$$

equation of tangent is $y - 0 = \frac{1}{2}(x-0)$

equation of normal is $y - 0 = -2(x - 0)$

9. Points of intersection of curve $x^2y = xy$ are

$(0, 1), (1, \frac{1}{2})$

The equation of tangent at (0, 1)

$$2xy + x^2y' = -y'$$

$$y' = 0$$

equation is $y - 1 = 0 \Rightarrow y = 1$

equation of tangent at $(1, \frac{1}{2})$ is $2xy + x^2y' = -y'$

$$1 + y' = -y' \Rightarrow y' = -\frac{1}{2}$$

$$y - \frac{1}{2} = -\frac{1}{2}(x - 1)$$

Put $y = 1$ $\quad \frac{1}{2} = -\frac{x}{2} + \frac{1}{2}$

$$x = 0$$

Point of intersection of tangent is (0, 1)

11. Let the point is (x_1, y_1)

Slope of line joining (0, 0) & (x_1, y_1) is

$$m_1 = \frac{y_1}{x_1}$$

$$\frac{(2x + 2yy')}{(x^2 + y^2)} = \frac{C\left(\frac{y'}{x} - \frac{y}{x^2}\right)}{\left(1 + \frac{y^2}{x^2}\right)}$$

$$\frac{2(x_1 + y_1y')}{(x_1^2 + y_1^2)} = \frac{C(y'x_1 - y_1)}{(x_1^2 + y_1^2)}$$

$$2x_1 + 2y_1y' = Cx_1y' - Cy_1$$

$$2x_1 + Cy_1 = y'(Cx_1 - 2y_1)$$

$$y' = \frac{(2x_1 + Cy_1)}{(Cx_1 - 2y_1)} = m_2$$

Calculate $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

16. $A + B + C = \pi \Rightarrow dA + dB = 0 \Rightarrow dA = -dB$

$\frac{c}{\sin C} = 2R$ = constant

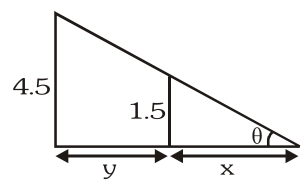
$a = 2R\sin A \Rightarrow da = 2R\cos A dA$ ....(i)

similarly $\quad db = 2R\cos B dB$ ....(ii)

Divide (i) by (ii)

$$\frac{da}{db} = \frac{\cos A(dA)}{\cos B(dB)} \Rightarrow \frac{da}{db} = -\frac{\cos A}{\cos B}$$

18.



$$\frac{4.5}{x+y} = \frac{1.5}{x} \Rightarrow 3x = x + y \Rightarrow 2x = y$$

$$\frac{2dx}{dt} = \frac{dy}{dt}$$

$\frac{dy}{dt} = 4$ is given then $\frac{dx}{dt} = 2$

(a) $\quad \frac{d}{dt}(x+y) = \frac{dx}{dt} + \frac{dy}{dt} = 4 + 2 = 6$ km/hr.

(b) $\quad$ Shadow lengthening $= \frac{dx}{dt} = 2$ km/hr.

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

2. $\frac{dv}{dt} = \frac{k}{r} \quad \Rightarrow \quad 4\pi r^2\frac{dr}{dt} = \frac{k}{r}$

$4\pi r^3 dr = k dt$

$\pi r^4 = kt + c$

at $\quad t = 0$; $r = 1 \quad \Rightarrow \quad c = \pi$

$$r^4 = \left(\frac{kt}{\pi} + 1\right)$$

put $\quad r = 2$ & $t = 15 \quad \Rightarrow \quad k = \pi$

$r^4 = t + 1$

$r = (t + 1)^{1/4}$

at $\quad t = 0$ volume $(v_1) = \frac{4}{3}\pi$

at time t volume $(v_2) = \frac{4}{3}\pi(t + 1)^{3/4}$

$v_2 = 27v_1$

$\Rightarrow \quad \frac{4}{3}\pi(t + 1)^{3/4} = (27)\frac{4}{3}\pi$

Solve for t

4. Mid point of AB is $\left(\frac{t_1+t_2}{2}, \frac{t_1^2+t_2^2}{2}\right)$

Equation of tangent at point A is

$y + t_1^2 = 2xt_1$ ... (i)

Equation of tangent at point B is

$y + t_2^2 = 2xt_2$ ... (ii)

(i) – (ii) $\Rightarrow$ $t_1^2 - t_2^2 = 2x(t_1 - t_2)$

$x = \frac{(t_1+t_2)}{2}$

$y + t_1^2 = t_1(t_1 + t_2)$

$y = t_1t_2$

Point C is $\left(\frac{(t_1+t_2)}{2}, t_1t_2\right)$

Length of median is $\left|\frac{t_1^2+t_2^2}{2} - t_1t_2\right|$

$= \frac{(t_1-t_2)^2}{2} = m$

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} t_1 & t_1^2 & 1 \\ t_2 & t_2^2 & 1 \\ \frac{t_1+t_2}{2} & t_1t_2 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$= \frac{1}{2}\begin{vmatrix} t_1 & t_1^2 & 1 \\ (t_2-t_1) & (t_2^2-t_1^2) & 0 \\ \frac{(t_2-t_1)}{2} & t_1(t_2-t_1) & 0 \end{vmatrix}$

$= \frac{(t_2-t_1)^2}{2}\begin{vmatrix} t_1 & t_1^2 & 1 \\ 1 & (t_2+t_1) & 0 \\ \frac{1}{2} & t_1 & 0 \end{vmatrix} = \frac{(t_2-t_1)^3}{4} = \frac{(2m)^{3/2}}{4}$

10. Any point on curve $y = x^2$ is $P(t, t^2)$

$\frac{dy}{dx} = 2x$

equation of normal at (t, t^2) is

$y - t^2 = -\frac{1}{2t}(x - t)$

Solving with $y = x^2$ we get

$x^2 - t^2 = \frac{-1}{2t}(x - t) \Rightarrow (x - t)\left(x + t + \frac{1}{2t}\right) = 0$

$\Rightarrow x = -t - \frac{1}{2t}$

So normal cuts the curve again at

$Q\left(-t - \frac{1}{2t}, \left(-t - \frac{1}{2t}\right)^2\right)$

$z = PQ^2 = 4t^2\left(1 + \frac{1}{4t^2}\right)^3$

Now $\frac{dz}{dt} = 0 \Rightarrow t = \pm\frac{1}{\sqrt{2}}, 0$

$\frac{dz}{dt}$ changes sign from negative to positive about

$t = \frac{1}{\sqrt{2}}$ as well as $t = -\frac{1}{\sqrt{2}}$

(No chord is formed for $t = 0$)

z is minimum at $t = \pm\frac{1}{\sqrt{2}}$ & minimum value of $z = PQ^2 = 3$

Shortest normal chord has length $\sqrt{3}$ & its equation is $x + \sqrt{2}y - \sqrt{2} = 0$

or $x - \sqrt{2}y + \sqrt{2} = 0$

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. $x = a(1 + \cos\theta)$, $y = a\sin\theta$

$\frac{dx}{d\theta} = -a\sin\theta$; $\frac{dy}{d\theta} = a\cos\theta$

$\left(\frac{dy}{dx}\right) = -\frac{\cos\theta}{\sin\theta}$ slope of normal $= -\left(\frac{dx}{dy}\right) = \frac{\sin\theta}{\cos\theta}$

$y - a\sin\theta = \frac{\sin\theta}{\cos\theta}(x - a - a\cos\theta)$

$y\cos\theta - a\sin\theta\cos\theta = x(\sin\theta) - a\sin\theta(1 + \cos\theta)$

$x\sin\theta - y\cos\theta = a\sin\theta(1 + \cos\theta - \cos\theta)$

clearly passes through (a, 0)

2. $x = a(\cos\theta + \theta\sin\theta)$ & $y = a(\sin\theta - \theta\cos\theta)$

$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$,

$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$

$\therefore \frac{dy}{dx} = \frac{\sin\theta}{\cos\theta}$

slope of normal $= -\frac{\cos\theta}{\sin\theta} = -\cot\theta$

it makes angle $\left(\frac{\pi}{2} + \theta\right)$ with the x-axis eq of normal $y - a\sin\theta + a\theta\cos\theta = -\dfrac{\cos\theta}{\sin\theta}$ $(x - a\cos\theta - a\theta\sin\theta)$

$\Rightarrow x\cos\theta + y\sin\theta = a.$

Hence it is at a constant distance 'a' from the origin.

3. Angle betweeen the tangents $\dfrac{dy}{dx} = 2x - 5$

$\left(\dfrac{dy}{dx}\right)_{(2,0)} = -1 \qquad \left(\dfrac{dy}{dx}\right)_{(3,0)} = 1 \Rightarrow \text{Angle} = \dfrac{\pi}{2}$

4. $y = x + \dfrac{4}{x^2}$

$\dfrac{dy}{dx} = 1 - \dfrac{8}{x^3}$

Equation of tangent is parallel to x-axis

$\therefore \dfrac{dy}{dx} = 0$

$\Rightarrow 1 - \dfrac{8}{x^3} = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$

At, $x = 2,\ y = 2 + \dfrac{4}{4} = 3 \Rightarrow y_1 = 3$

$\therefore$ point is (2, 3)

equation of tangent is :

$y - y_1 = 0(x - x_1)$

$y = 3$

5. $y = \int_0^x |t|\,dt$

$\dfrac{dy}{dx} = |x| = 2 \qquad \Rightarrow \qquad x = \pm 2$

If $x = 2$, $\quad y = \int_0^2 t\,dt = 2$

If $x = -2$, $\quad y = \int_0^{-2} -t\,dt = -2$

Tangents are $(y - 2) = 2(x - 2)$ or

$(y + 2) = 2(x + 2)$

x intercepts $= \pm 1$.

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. Slope of normal

$= -\dfrac{1}{dy/dx} = \tan\dfrac{3\pi}{4} \Rightarrow \dfrac{dy}{dx} = 1$

$\therefore f'(3) = 1$

2. $3y^2y' + 6x = 12y'$

$2x = y'(4 - y^2)$

$y' = \dfrac{2x}{(4 - y^2)}$

For vertical tangent $y = \pm 2$

At $y = 2$

$8 + 3x^2 = 24 \Rightarrow 3x^2 = 16 \Rightarrow x = \pm\dfrac{4}{\sqrt{3}}$

At $y = -2$

$-8 + 3x^2 = -24$

x^2 = negative

Not possible

4. Put $x_1 = x + h$ & $x_2 = x$

$|f(x + h) - f(x)| \le h^2$

$\lim_{h\to 0}\left|\dfrac{f(x+h) - f(x)}{h}\right| \le \lim_{h\to 0} h$

$|f'(x)| \le 0$

Possible only if $f'(x) = 0$

$f(x) = c$

at point (1, 2) $\quad f(x) = 2$

$y = 2$