QUADRATIC EQUATION

EXERCISE - 01 CHECK YOUR GRASP

1. Since sum of coefficients = 0

$\therefore$ It's one root is 1 and other root is $\frac{a-2b+c}{a+b-2c}$

2. Hint : $\frac{\alpha+\beta}{2}=\frac{8}{5}$ and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{16}{7}$

5. $\tan\left(\frac{\pi}{12}\right)=2-\sqrt{3}$

$\Rightarrow$ other root = $2+\sqrt{3}$

9. $q^2 - 4p \geq 0$

$q = 2 \Rightarrow p = 1$

$q = 3 \Rightarrow p = 1, 2$

$q = 4 \Rightarrow p = 1, 2, 3, 4$

Hence 7 values of (p, q)

7 equations are possible.

11. $b^2 - 4ac < 0$

Now $a^2x^2 + abx + ac$

$\Rightarrow$ $D = (ab)^2 - 4(a^2)(ac) = a^2(b^2 - 4ac)$

Here $D < 0$

also coefficient of x^2 is positive.

curve is always above x axis

$\Rightarrow$ expression is always positive.

13. Hint : A = 1, B = 7, C = 12

17. Let x – b be the common factor

$\Rightarrow$ $b^2 - 11b + a = 0$ ....(i)

Also $b^2 - 14b + 2a = 0$ ....(ii)

from (i) and (ii)

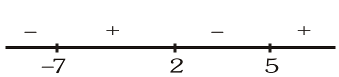
$\Rightarrow$ $-a - 3b + 2a = 0$

$\Rightarrow$ $a = 3b$ $\Rightarrow$ $b = \frac{a}{3}$

Put $b=\frac{a}{3}$ in equation : $\frac{a^2}{9}-\frac{11a}{3}+a=0$

$\Rightarrow$ a = 24

18. $\frac{x-5}{x^2+5x-14} > 0$ $\Rightarrow$ $\frac{(x-5)}{(x+7)(x-2)}>0$



smallest integer is – 6

21. $D > 0$

$\Rightarrow$ $36 - 4b > 0$ $\Rightarrow$ $b < 9$

Also $|\alpha-\beta|\leq 4$

$\Rightarrow$ $\sqrt{(\alpha+\beta)^2-4\alpha\beta}\leq 4$ $\Rightarrow$ $\sqrt{36-4b}\leq 4$

$\Rightarrow$ $\sqrt{9-b}\leq 2$ $\Rightarrow$ $9 - b \leq 4$

$\Rightarrow$ $b \geq 5$

least value = 5

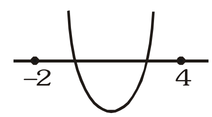
22. $\frac{x^2+2x+1}{x^2+2x+7}=y$

$\Rightarrow$ $x^2(1-y)+x(2-2y)+1-7y=0$

$D > 0$

$\Rightarrow$ $6y(y - 1) < 0$ $\Rightarrow$ $y \in [0, 1)$

23. Hint : $x^2 - 2mx + m^2 - 1 = 0$



(i) f (–2) > 0

(ii) f (4) > 0

(iii) $D \geq 0$

(iv) $-2 < \frac{-b}{2a} < 4$

Common solution $m \in (-1, 3)$

24. Let roots of $x^3 - Ax^2 + Bx - C = 0$ are α, β, γ

$\Rightarrow \alpha + \beta + \gamma = A$, $\Sigma\,\alpha\beta = B$, $\alpha\beta\gamma = C$

& $(\alpha + 1)(\beta + 1)(\gamma + 1) = 19$

$\Rightarrow (\alpha + \beta + \gamma) + (\alpha\beta + \beta\gamma + \gamma\alpha) + \alpha\beta\gamma + 1 = 19$

A + B + C = 18

25. $\frac{\sum\alpha\beta\gamma}{\alpha\beta\gamma\delta}=\frac{-4}{10}=\frac{-2}{5}$

26. $(x^2 + 4)^2 = (2x - 3)^2$

$\Rightarrow$ $x^2 + 4 = \pm(2x - 3)$

$\Rightarrow$ $x^2 + 2x + 1 = 0$ or $\underbrace{x^2-2x+7}_{D<0}=0$

$\Rightarrow$ $(x + 1)^2 = 0$ or No solution

$\Rightarrow$ $x = -1$

Have only one solution.

36. $\frac{1}{2}\leq \log_{1/10} x \leq 2$

$\Rightarrow$ $\left(\frac{1}{10}\right)^{1/2}\geq x$, $\left(\frac{1}{10}\right)^{2}\leq x$

So, $\frac{1}{100}\leq x\leq\frac{1}{\sqrt{10}}$

2. Hint : $\cos\alpha = \dfrac{-5 \pm \sqrt{25 + 4 \times 12 \times 25}}{50} = -\dfrac{4}{5}, \dfrac{3}{5}$

3. $x^2 + (p + q)x + pq = r(2x + p + q)$

$\Rightarrow \quad x^2 + (p+q - 2r)x + pq - rp - rq = 0$

sum of roots = 0 $\Rightarrow p + q - 2r = 0$

$\Rightarrow \quad p + q = 2r$

product of roots $= pq - r(p + q) = pq - \dfrac{(p+q)^2}{2}$

$= -\dfrac{1}{2}(p^2 + q^2)$

5. $D = 25b^2 - 84ac = 25(a + c)^2 - 84ac$

$(\because a + b + c = 0)$

$= 4(a + c)^2 + 21(a - c)^2 > 0$

7. $\alpha + \beta + \gamma = 0, \quad \Sigma\, \alpha\beta = \dfrac{b}{a}, \quad \alpha\beta\gamma = -\dfrac{c}{a}$

Also $\alpha + \beta = -P, \quad \alpha\beta = 1 \Rightarrow \quad \gamma = -\dfrac{c}{a}$ put in eqn.

$-\dfrac{c^3}{a^3}\cdot a - b\cdot\dfrac{c}{a} + c = 0 \quad \Rightarrow \quad \dfrac{c^2}{a^2} + \dfrac{b}{a} - 1 = 0$

$\Rightarrow \quad c^2 + ab - a^2 = 0 \quad \Rightarrow a^2 - c^2 = ab$

8. $(x - 3a)(x - (a + 3)) < 0 \quad \Rightarrow \quad 3a < x < a + 3$

Now $\quad 1 \le x \le 3 \Rightarrow$ min. at $\quad 3a < 1 \Rightarrow a < \dfrac{1}{3}$

max. at $\Rightarrow \quad 3 < a + 3 \quad \Rightarrow \quad a > 0 \quad \Rightarrow \quad a \in \left(0, \dfrac{1}{3}\right)$

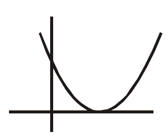
Reverse $a + 3 < x < 3a \Rightarrow a + 3 < 1 \Rightarrow a < -2$

$3 < 3a \quad \Rightarrow \quad a > 1$ no solution

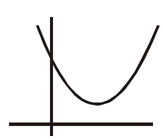
10. Put $x = 0$

we get $f(x) = f(0) > 6$

graph is shown as



$\Rightarrow \quad f(2) \ge 0$



$4a + 2b + 6 \ge 0$

$2a + b \ge -3$

$\Rightarrow$ least value $= -3$

11. Given expression reduce to

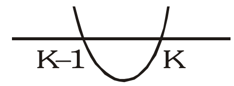
$[-(p - q) - (x - p) + (x - q)]^2 = 0$

$\Rightarrow \quad [-p + q - x + p + x - q]^2 = 0$

hence $x \in R$.

12. Hint : $D = 0$

13. (A)



Expression is $(x^2 + (K+K-1)x + K(K-1) = 0$

$\Rightarrow \quad (x - K)(x - K + 1) = 0$

$\Rightarrow \quad x = K$ or $x = K - 1$

$\Rightarrow$ greater part $< 2 \quad \Rightarrow K < 2$

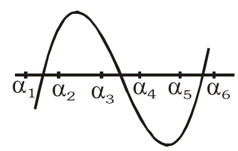
(B) Opposite sign $\left(\dfrac{c}{a} < 0\right) \Rightarrow K(K-1) < 0$

$\Rightarrow \quad K \in (0,1)$

(C) $K(K-1) > 0 \Rightarrow K \in (-\infty, 0) \cup (1, \infty)$

(D) $K - 1 > 2 \quad \Rightarrow K > 3 \Rightarrow K \in (3, \infty)$

15.



$\left.\begin{array}{l} f(\alpha_1) = -ve \\ f(\alpha_2) = +ve \end{array}\right]$ one root $\in (\alpha_1, \alpha_2)$

$\left.\begin{array}{l} f(\alpha_3) = +ve \\ f(\alpha_4) = -ve \end{array}\right]$ one root in (α_3, α_4)

$\left.\begin{array}{l} f(\alpha_5) = - \\ f(\alpha_6) = + \end{array}\right]$ one root in (α_5, α_6)

17. $\log_{\frac{1}{5}}(bx + 28) = \log_{\frac{1}{5}}(12 - 4x - x^2)$

$\Rightarrow \quad bx + 28 = 12 - 4x - x^2$

$x^2 + (b + 4)x + 16 = 0$

$\Rightarrow$ since $D = 0 \qquad (b + 4)^2 - 64 = 0$

$\Rightarrow \quad b = 4, -12$

At $\quad b = -12, \quad x = 4$

Put in log domain $\Rightarrow$ No solution

at $b = 4$ satisfies the domain

18. $x^5(x^3 - 1) + x(x - 1) + 1 = y$

when $x \ge 1 \qquad y > 0$

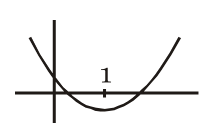
$-1 < x < 1 \qquad y > 0$

$x \le -1 \qquad y > 0$

hence $y > 0$

19. $p^2 + p + 1 = a$

is always positive



$\Rightarrow \quad af(1) < 0$

20. $x^4 - px^3 + qx^2 - rx + s = 0$

$\Rightarrow \quad \tan(A + B + C + D) = \tan(\pi + D)$

$\Rightarrow \quad \tan D = \dfrac{s_1 - s_3}{1 - s_2 + s_4} = \dfrac{p - r}{1 - q + s}$

True & False :

3. $D = \cos^2 p - 4\sin p(\cos p - 1)$

$= \cos^2 p + 4\sin p(1 - \cos p) > 0$

Fill in the blanks :

3. $a\alpha^2 + b\alpha + c = 0$ & $a\beta^2 + b\beta + c = 0$

$\therefore \dfrac{a\alpha^2}{-a\alpha^2} + \dfrac{a\beta^2}{-a\beta^2} = -2$

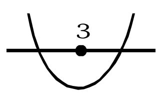
Match the Column :

1. $x^2 + 2(a - 1)x + a + 5 = 0$

$D = 4(a - 1)^2 - 4(a + 5)$

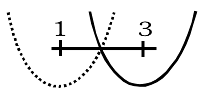
$= 4(a^2 - 3a - 4) = 4(a - 4)(a + 1)$

(A) $D < 0 \Rightarrow a \in (-1, 4)$

(B) 

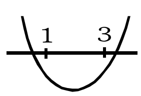
$\Rightarrow f(3)<0$

$\Rightarrow 7a + 8 < 0 \Rightarrow a \in \left(-\infty, -\dfrac{8}{7}\right)$

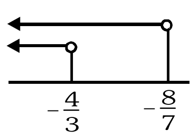
(C) 

$\Rightarrow f(1) \,.\, f(3) < 0$

$(3a + 4)(7a + 8) < 0 \quad a \in \left(-\dfrac{4}{3}, -\dfrac{8}{7}\right)$

(D) 

$\Rightarrow f(1) < 0$ & $f(3) < 0$



$\Rightarrow a \in \left(-\infty, -\dfrac{4}{3}\right)$.

Assertion & Reason :

2. $f(x) = a(x + 1)(x - \beta)$ (as -1 is root)

$f(1) + f(2) = 2a(1 - \beta) + 3a(2 - \beta) = 0$

$= a(8 - 5\beta) = 0$ as $a \neq 0 \Rightarrow \beta = \dfrac{8}{5}$

4. $a_1 + a_2 + a_3 + a_4 + a_5 = 0$

$\Rightarrow$ 1 is one of the root of the equation & degree of equation is 4 & complex roots occur in conjugate

$\Rightarrow$ at least 2 real roots.

Comprehension # 1 :

1. $a\left(\dfrac{2x+1}{x-1}\right)^2 + b\left(\dfrac{2x+1}{x-1}\right) + c = 0$

Let $\dfrac{2x+1}{x-1} = \alpha$

$\Rightarrow x\alpha - \alpha = 2x + 1$

$\Rightarrow x(\alpha - 2) = 1 + \alpha \Rightarrow x = \dfrac{1+\alpha}{\alpha-2}$

Similarly other root can be find out.

2. $\dfrac{1}{2\alpha-3} = x \Rightarrow 2\alpha - 3 = \dfrac{1}{x} \Rightarrow \alpha = \dfrac{1}{2}\left(\dfrac{1}{x}+3\right)$

$\Rightarrow 2\left(\dfrac{1+3x}{2x}\right)^2 + 4\left(\dfrac{1+3x}{2x}\right) - 5 = 0$

$\Rightarrow 11x^2 + 10x + 1 = 0.$

4. $x = \dfrac{1+\alpha}{1-\alpha} \Rightarrow \alpha = \dfrac{x-1}{x+1}$

$\Rightarrow \left(\dfrac{x-1}{x+1}\right)^3 - \left(\dfrac{x-1}{x+1}\right) - 1 = 0$

$\Rightarrow x^3 + 7x^2 - x + 1 = 0 \Rightarrow \Pi\left(\dfrac{1+\alpha}{1-\alpha}\right) = -1.$

Comprehension # 2 :

1. $(2-\sqrt{3})(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1}(2-\sqrt{3})$

$= 4$

$\Rightarrow (2+\sqrt{3})^{x^2-2x} + (2-\sqrt{3})^{x^2-2x} = 4$

$\Rightarrow x^2 - 2x = \pm 1 \Rightarrow x = 1 \mp \sqrt{2}, 1$

2. $\Rightarrow x^2 - 2|x| = \pm 1$

$\Rightarrow |x| = 1 \pm \sqrt{2}, 1$ but $|x| \neq 1 - \sqrt{2}$

$\Rightarrow x = \pm(1 + \sqrt{2}), \pm 1$ 4 solution.

3. $\sqrt{a\sqrt{a\sqrt{a\ldots\ldots\ldots}}} = a^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ldots\ldots} = a^{\frac{1/2}{1-1/2}}$

$= a = x^2 - 3$

also $\sqrt{49+20\sqrt{6}} = \sqrt{25+20\sqrt{6}+24}$

$= \sqrt{(5+2\sqrt{6})^2} = (5+6\sqrt{2})$

hence $(5+2\sqrt{6})^{x^2-3} + (5+2\sqrt{6})^{x^2+x-3-x} = 10$

$x^2 - 3 = \pm 1 \Rightarrow x = \pm 2$ & $x = \pm\sqrt{2}$

domain of $\sqrt{a} \Rightarrow x^2 - 3 > 0 \Rightarrow x > \sqrt{3}$

OR $x < -\sqrt{3}$

hence $x = 2$ only solution.

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

3. $D_1 = p^2 - 4q$ & $D_2 = r^2 - 4s$

Add $p^2 + r^2 - 4(q + s) = D_1 + D_2$

$= p^2 + r^2 - 2pr = (p - r)^2 \geq 0$

$\Rightarrow D_1 + D_2 \geq 0$ possible only if at least one of D is ≥ 0

5. $x^2 + 18x + 45 - 2\sqrt{x^2 + 18 + 45} + 1 = 16$

$\Rightarrow \left(\sqrt{x^2 + 18x + 45} - 1\right)^2 = 16$

$\Rightarrow \sqrt{x^2 + 18x + 45} - 1 = \pm 4$

$\Rightarrow \sqrt{x^2 + 18x + 45} = \pm 4 + 1 = 5, -3$

$\Rightarrow x^2 + 18x + 45 = 25$, (Reject -3)

$\Rightarrow x^2 + 18x + 20 = 0$

Product of root = +20.

6. Considering denominator $x^2 - 8x + 32$

$D < 0$ and $a > 0$

So denominator is always positive

$\Rightarrow ax^2 + 2(a + 1)x + 9a + 4 < 0$

$\Rightarrow a < 0$ & $4(a + 1)^2 - 4a(9a + 4) < 0$

$\Rightarrow 4(a^2 + 2a + 1 - 9a^2 - 4a) < 0$

$\Rightarrow 4(-8a^2 - 2a + 1) < 0$

$8a^2 + 2a - 1 > 0$

$(4a - 1)(2a + 1) > 0$

$\Rightarrow a \in \left(-\infty, -\frac{1}{2}\right)$

7. Hint : $\frac{x^2 + ax - 2}{x^2 + x + 1} + 3 > 0 \Rightarrow \frac{4x^2 + (a + 3)x + 1}{x^2 + x + 1} > 0$

as Denominator $> 0 \Rightarrow$ Numerator > 0

and $\frac{x^2 + ax - 2}{x^2 + x + 1} - 2 < 0 \Rightarrow \frac{-x^2 + (a - 2)x - 4}{x^2 + x + 1} < 0$

as Denominator $> 0 \Rightarrow$ Numerator < 0

Now solve it.

9. Let $x^2 - ax + b = 0$ has root α & β

& $x(x^2 - px + q) = 0$ has root $0, \alpha, \alpha$

but $x = 0$ is not the common root (since $b \neq 0$)

& $x^2 - px + q = 0$ have two equal root

$\Rightarrow D = 0 \Rightarrow p^2 - 4q = 0$ & $\frac{p}{2}$ is its root.

$\frac{p}{2}$ satisfy equation $x^2 - ax + b = 0$

$\Rightarrow \frac{p^2}{4} - \frac{ap}{2} + b = 0 \Rightarrow \frac{p^2}{4} + b = \frac{ap}{2}$

$\Rightarrow 2(q + b) = ap$

11. Let $x^2 - x = t$

$(t - 1)(t - 7) + 5 < 0 \Rightarrow t^2 - 8t + 12 < 0$

$\Rightarrow (t - 6)(t - 2) < 0 \Rightarrow 2 < x^2 - x < 6$

$x^2 - x - 6 < 0$ & $x^2 - x > 2$

$(x - 3)(x + 2) < 0$ & $(x - 2)(x + 1) > 0$

$\Rightarrow x \in (-2, -1) \cup (2, 3)$

12. $(2x - 2)\left(\frac{(x^2 - 2x)^2 - 9}{x^2 - 2x}\right) \leq 0$

$\Rightarrow 2(x - 1)\left(\frac{(x^2 - 2x + 3)(x^2 - 2x - 3)}{x(x - 2)}\right) \leq 0$

$\Rightarrow 2(x - 1)\frac{(x^2 - 2x + 3)(x - 3)(x + 1)}{x(x - 2)} \leq 0$

$\Rightarrow x \in (-\infty, -1] \cup (0, 1] \cup (2, 3]$

23. Hint : $D \geq 0$ & $f(0) . f(3) > 0$ & $0 < (-b/2a) < 3$.

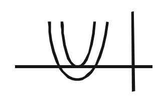
24. $x^2 + 2(K - 1)x + K + 5 = 0$

$D > 0$

$\Rightarrow 4(K^2 - 3K - 4) \geq 0 \Rightarrow (K - 4)(K + 1) \geq 0$

$\Rightarrow K \in (-\infty, -1) \cup (4, \infty)$

Now subtract those cases in



which both roots are negative

(i) $f(0) > 0 \Rightarrow K + 5 > 0 \Rightarrow K > -5$

(ii) $-\frac{b}{2a} < 0 \Rightarrow 1 - K < 0 \Rightarrow K > 1$

So for $K > 1$ boths root are negative

hence for atleast one positive root $K \in (-\infty, -1)$

25. Let $y = (x - a)(x - c) + 2(x - b)(x - d)$

then $y(a) > 0$ $y(c) < 0$

$y(b) < 0$ $y(d) > 0$

y have one real root between a & b

& one real root between c & d

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

3. Let $y = 2^{\cos^2 x} \Rightarrow y + \frac{1}{y} = \frac{2}{p}$

$\because \cos^2 x \geq 0 \Rightarrow y \geq 1$

$\therefore y + \frac{1}{y} \geq 2 \Rightarrow \frac{2}{p} \geq 2 \Rightarrow p \leq 1$

Also $2^{\cos^2 x} = y \leq 2 \Rightarrow \frac{2}{p} \leq 2 + \frac{1}{2}$

$\Rightarrow p \geq \frac{4}{5} \Rightarrow p \in \left[\frac{4}{5}, 1\right]$

4. Let the roots are rational

$a = 2\ell + 1 \quad b = 2m + 1 \quad c = 2n + 1$

then $D = (2m+1)^2 - 4(2\ell + 1)(2n + 1)$

$= \text{odd}^2 - 4 \text{ odd odd} = \text{odd}^2 - \text{even} = \text{say}(2p + 1)^2$

$\Rightarrow (2m + 1)^2 - 4(2\ell + 1)(2n + 1) = (2p + 1)^2$

$\Rightarrow (2m + 1)^2 - (2p + 1)^2 = 4(2\ell + 1)(2n + 1) = \text{even}$

$\Rightarrow (m - p)(m + p + 1) = (2\ell + 1)(2n + 1)$

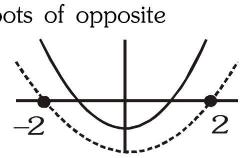
Case-I : m is odd p is even

$\left.\begin{array}{l}\text{LHS = odd even = even}\\ \text{RHS = odd}\end{array}\right\}$ Not possible

$\left.\begin{array}{l}\text{Similarly for m even p odd}\\ \text{m even p even}\\ \text{m odd p odd}\end{array}\right\}$ do not hold

hence roots can not be rational

6. $ax^2 + bx + c = 0$ have real roots of opposite sign in $(-2, 2)$



$\Rightarrow x^2 + \frac{b}{a} x + \frac{c}{a} = 0$

(1) $D \geq 0 \Rightarrow \frac{b^2}{a^2} - \frac{4c}{a} \geq 0$

(2) $f(-2) > 0 \Rightarrow 4 - \frac{2b}{a} + \frac{c}{a} > 0 \Rightarrow 1 + \frac{c}{4a} - \frac{b}{2a} > 0$

(3) $f(2) > 0 \Rightarrow 4 + \frac{2b}{a} + \frac{c}{a} > 0 \Rightarrow 1 + \frac{c}{4a} + \frac{b}{2a} > 0$

(4) $\alpha\beta = \frac{c}{a} - 4 < \frac{c}{a} < 0 \Rightarrow 1 + \frac{c}{4a} > 0$

(5) $-2 < -\frac{b}{2a} < 2$

combined condition from (2) & (3)

$\Rightarrow 1 + \frac{c}{4a} - \left|\frac{b}{2a}\right| > 0$

7. $z = x^2 + y^2 + 1 + 2xy + 2x + 2y + x^2 - 4x + 4 - 4 + 1$

$= (x + y + 1)^2 + (x - 2)^2 - 3$

minimum value of z is -3

9. $x^n + px^2 + qx + r = 0 \quad n \geq 5$

(a) $S_1 \equiv \alpha_1 + \alpha_2 + \ldots\ldots\ldots + \alpha_n$

$S_2 = \alpha_1^2 + \alpha_2^2 + \ldots\ldots\ldots + \alpha_n^2$

$= (\alpha_1 + \alpha_2 + \ldots\ldots\alpha_n)^2 - 2(\Sigma\alpha_1\alpha_2)$

$= 0$ (as coefficient of x^{n-1}, $x^{n-2} = 0$)

as $\alpha_1^2 + \alpha_2^2 + \ldots\ldots + \alpha_n^2 = 0$

$\Rightarrow \alpha_1 = \alpha_2 = \alpha_3 \ldots = \alpha_n = 0$

but $\prod_{i=1}^{n} \alpha_i = (-1)^n r \neq 0$

hence all of roots are not real

(b) $S_n + pS_2 + qS_1 + nr = 0$

$\Rightarrow (\alpha_1^n + p.\alpha_1^2 + q.\alpha_1 + r)$

$+ (\alpha_2^n + p.\alpha_2^2 + q.\alpha_2 + r) + \ldots$

$\ldots + (\alpha_n^n + p.\alpha_n^2 + q.\alpha_n + r)$

$= 0 + 0 + \ldots\ldots + 0 = 0$

$\Rightarrow S_n = -pS_2 - qS_1 - nr = 0 + 0 - nr$

$\Rightarrow S_n = -nr$

10. $2\log_{\frac{1}{25}}(bx + 28) = \log_{\frac{1}{5}}(12 - 4x - x^2)$

$\Rightarrow bx + 28 = 12 - 4x - x^2$

$\Rightarrow x^2 + (b + 4)x + 16 = 0$

for only one solution

Case-I :

$D = 0$ or expression is perfact square $\Rightarrow b = 4, -12$

at $b = 4 \Rightarrow x = -4$ satisfies both log domain

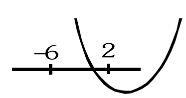
at $b = -12 \Rightarrow x = 4$ do not satisfies the log domain

$\Rightarrow b = 4$

domain $12 - 4x - x^2 > 0 \Rightarrow x^2 + 4x - 12 < 0$

$\Rightarrow -6 < x < 2$ & $bx + 28 > 0$

Case-II : Only one root in domain of log



$\Rightarrow f(-6) . f(2) < 0$

$\Rightarrow (28 - 6b)(28 + 2b) < 0$

$\Rightarrow b \in (-\infty, 14) \cup (14/3, \infty)$

at $x = -6$ at $b = \frac{14}{3}$, x still satisfy the domain

similarily at $x = 2 \Rightarrow b = -14$

x do not satisfies the domain.

$\Rightarrow b \in (-\infty, -14) \cup \{4\} \cup \left[\frac{14}{3}, \infty\right)$.

2. $(x-a)(x-b)-c=(x-\alpha)(x-\beta)$

$(x-\alpha)(x-\beta)+c=(x-a)(x-b)$

so $(x-\alpha)(x-\beta)+c=0$ have roots a, 6

4. Let roots $\alpha, 2\alpha$

$$3\alpha = \frac{3a-1}{a^2-5a+3}$$

$$2\alpha^2 = \frac{2}{a^2-5a+3}$$

$$\frac{2\alpha^2}{9\alpha^2} = \frac{2}{a^2-5a+3}\ \frac{(a^2-5a+3)^2}{(3a-1)^2}$$

$$\frac{2}{9} = \frac{2(a^2-5a+3)}{(3a-1)^2}$$

$9a^2-45a+27=9a^2-6a+1$

$39a=26$

$$\boxed{a=\frac{2}{3}}$$

5. $\alpha+\beta=\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$ given

$$\alpha+\beta=\frac{(\alpha^2+\beta^2)}{\alpha^2\beta^2}$$

$$(\alpha+\beta)=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$$

$$\frac{-b}{a}=\frac{b^2-2ac}{c^2}$$

$-bc^2=ab^2-2a^2c \Rightarrow bc^2+ab^2=2a^2c$

$$\frac{c}{a}+\frac{b}{c}=\frac{2a}{b}$$

so

$\dfrac{c}{a}, \dfrac{a}{b}, \dfrac{b}{c}$ … A.P.

$\dfrac{a}{c}, \dfrac{b}{a}, \dfrac{c}{b}$ … H.P.

7. $x+\dfrac{1}{x}\ge 2$

$\because$ AM $\ge$ GM

$x+\dfrac{1}{x}$ is min at $x=1$

9. $x^2+px+(1-p)=0$

$(1-p)^2+p(1-p)+(1-p)=0$

$(1-p)(1-p+p+1)=0$

$p=1$

$x^2+x=0$

$x=0, -1$

10. $x^2+px+12=0$

$16+4p+12=0$

because 4 is root

$\boxed{p=-7}$

$x^2+px+q=0$ has equal root

$p^2=4q$

$49=4q$ $\boxed{q=\frac{49}{4}}$

11. $x^2-(a-2)x-a-1=0$

$a^2+b^2=(\alpha+\beta)^2-2\alpha\beta$

$(a-2)^2+2a+2$

a^2-2a+6

$(a-1)^2+5$ is min. at

$\boxed{a=1}$

12. For consecutive integers roots

$|\alpha-\beta|=1$

$b^2-4c=1$

14. $f(x)=a_nx^n+a_{n-1}x^{n-1}+\dots\dots+a_1x=0$

$f(0)=0=f(\alpha)$

so according to Roll's theorem

$f'(x)=0$ have at least one root $(0,\alpha)$

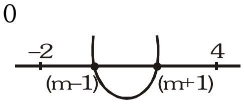
so root of equation

$na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\dots\dots a_1=0$

has roots less than α.

15. $x^2-2mx+m^2-1=0$

$(x-m)^2=1$



$x=m\pm 1$

$m+1<4$ and $m-1>-2$

$m<3$ and $m>-1$

so $\boxed{m\in=(-1,3)}$

17. $y=\dfrac{3x^2+9x+7+10}{3x^2+9x+7}$

$$y=1+\frac{10}{3x^2+9x+7}$$

$$y=1+\frac{10}{p}$$

p is min then y max

$p=3x^2+9x+7$

$$p_{min} = \frac{-D}{4a} = \frac{-(81-12\times 7)}{12}$$

$$p_{min} = \frac{1}{4}$$

$$y_{max} = 1 + \frac{10}{1/4} = 41$$

19. $x^2 - 6x + a = 0$ $\quad \alpha, \beta$

$x^2 - cx + 6 = 0$ $\quad \alpha, \gamma$

let common root α and $\beta\ \gamma \rightarrow$ integer

$$\frac{\beta}{\gamma} = \frac{4}{3}$$

$\alpha\beta = a$

$\alpha\gamma = 6$

$$\frac{\beta}{\gamma} = \frac{a}{6} = \frac{4}{3}$$

$\boxed{a = 8}$

root of (1) equation

$x^2 - 6x + 8 = 0$

$x = 2, 4$

β can't be equal to 2

because at $\beta = 2$ $\quad \gamma = \frac{3}{2}$

which is not integer

so $\beta = 4$ and $\alpha = 2$

common root $\boxed{\alpha = 2}$

22. $P(x) = k(x + 1)^2$

$P(-2) = 2 = k(-1)^2$

$\Rightarrow k = 2$

$\therefore$ $P(x) = 2(x + 1)^2$

$\Rightarrow P(2) = 18$

Aliter :

$P(x) = (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0$

only $x = -1$ $P(x) = 0$

So roots are equal.

it means $\quad D = 0$

$(b - b_1)^2 = 4(a - a_1)(c - c_1) \Rightarrow q^2 = 4pr$

Let $\quad a - a_1 = p$

$\quad b - b_1 = q$

$\quad c - c_1 = r$

$P(-1) = 0$

$p - q + r = 0$ ...... (1)

$4p - 2q + r = 2$ ..... (2)

$4p + 2q + r = ?$

From (1) $\quad q = p + r$

$(p + r)^2 - 4pr = 0$

$(p - r)^2 = 0$ $\quad \boxed{p = r}$

from eq. (1) $q = 2r$

So from eq. (2) $\quad 4r - 4r + r = 2$

$r = 2$

So $4p + 2q + r = 4r + 4r + r = 9r = 18$

24. Given $e^{\sin x} - e^{-\sin x} = 4$

let $\quad e^{\sin x} = y$

$$y - \frac{1}{y} = 4 \quad \Rightarrow \quad y^2 - 4y - 1 = 0$$

$$y = 2 \pm \sqrt{5}$$

$e^{\sin x} = 2 + \sqrt{5}$ $\qquad$ $e^{\sin x} = 2 - \sqrt{5}$

but we know that

$$e^{-1} \le e^{\sin x} \le e^{1}$$

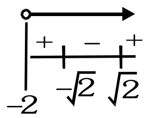
so $\quad e^{\sin x} \ne 2 + \sqrt{5}$ and $2 - \sqrt{5}$

so No real solution of given equation.

2. $x^2 - |x+2| + x > 0$

Case-I : $x + 2 \ge 0 \Rightarrow x^2 - x - 2 + x > 0$

$\Rightarrow$ $x^2 - 2 > 0$



$\Rightarrow$ $x \in \left[-2, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

Case-II : $x + 2 < 0$

$x^2 + x + 2 + x > 0 \quad \Rightarrow \quad x^2 + 2x + 2 > 0$

$\Rightarrow$ $x < -2$ is solution

$\Rightarrow$ $\left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

6. (b) $x^2 - 10\,cx - 11d = 0 \begin{matrix} a \\ b \end{matrix}$

$x^2 - 10ax - 11b = 0 \begin{matrix} c \\ d \end{matrix}$

$a + b = 10c$ ......(i)

& $c + d = 10a$ ......(ii)

add (i) & (ii)

$\Rightarrow$ $a + b + c + d = 10(a + c)$

subract (i) & (ii)

$(a - c) + (b - d) = 10(c - a)$

$\Rightarrow$ $b - d = 11(c - a)$ ......(iii)

also $a^2 - 10ca - 11d = 0$ ......(iv)

$c^2 - 10ac - 11b = 0$ ......(v)

from (iv) & (v)

$\Rightarrow$ $a^2 - c^2 = 11(d - b)$

$(a - c)(a + c) = 11(d - b)$

$\Rightarrow$ $(a + c) = 121$ (from (iii))

and $a + b + c + d = 10(a + c)$

$= 121 \quad 10 = 1210$

7. (a) $x^2 - px + r = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$ $x^2 - qx + r \begin{matrix} \alpha/2 \\ 2\beta \end{matrix}$

$\alpha + \beta = p$, $\dfrac{\alpha}{2} + 2\beta = q \Rightarrow \alpha + 4\beta = 2q$

$\alpha\beta = r \qquad \alpha\beta = r$

$\Rightarrow$ $3\beta = (2q - p)$

$\Rightarrow$ $\beta = \dfrac{2q - p}{3}$ and

$\alpha = p - \dfrac{(2q - p)}{3} = \dfrac{4p - 2q}{3}$

$r = \alpha\beta = \dfrac{2}{9}(2p - q)(2q - p)$

8. $x^2 + 2px + q = 0$

then $\alpha + \beta = -2p$ & $\alpha\beta = q$

and $ax^2 + 2bx + c = 0$

$\alpha + \dfrac{1}{\beta} = -\dfrac{2b}{a}$ & $\dfrac{\alpha}{\beta} = \dfrac{c}{a}$

$\dfrac{\alpha\beta + 1}{\beta} = \dfrac{-2b}{a} \Rightarrow \dfrac{q + 1}{\beta} = -\dfrac{2b}{a}$

$\Rightarrow$ β is real $\Rightarrow$ α is real

so $(p^2 - q)(b^2 - ac) \ge 0$

hence S(I) is true

Let $\dfrac{b}{a} = p$ and $\dfrac{c}{a} = q$

$x^2 + \dfrac{2b}{a}x + \dfrac{c}{a} = 0 \Rightarrow x^2 + 2px + q = 0$

$\Rightarrow$ $\beta = \dfrac{1}{\beta} \Rightarrow \beta = \pm 1$ (not possible)

Hence S(II) is True but S(II) is not the correct explaination of S(I)

10. $\alpha^3 + \beta^3 = q$

$\Rightarrow$ $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = q$

$\Rightarrow$ $\alpha\beta = \dfrac{p^3 + q}{3p}$

sum of the roots $= \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$

$$= \frac{p^2 - 2\left(\dfrac{p^3 + q}{3p}\right)}{\dfrac{p^3 + q}{3p}} = \frac{p^3 - 2q}{p^3 + q}$$

Product of the roots = 1.

Required equation is

$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

11. α, β are roots of $x^2 - 6x - 2 = 0$

$\Rightarrow$ $\alpha^2 - 6\alpha - 2 = 0$ & $\beta^2 - 6\beta - 2 = 0$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8.6\alpha - \beta^8.6\beta}{2(\alpha^9 - \beta^9)} = 3$$