

# PROBABILITY

## EXERCISE - 01

## CHECK YOUR GRASP

1. Let the roots of the quadratic equation be  $\alpha, \beta$   
 After squaring  $\alpha^2, \beta^2$   
 $\alpha\beta = (\alpha\beta)^2 \Rightarrow \alpha\beta(\alpha\beta - 1) = 0$   
 $\Rightarrow \alpha\beta = 0 \quad \dots (1)$   
 $\Rightarrow \alpha\beta = 1 \quad \dots (2)$   
 Now  $\alpha^2 + \beta^2 = \alpha + \beta$   
 $(\alpha + \beta)^2 - 2\alpha\beta = (\alpha + \beta)$   
 $\Rightarrow (\alpha + \beta)^2 - (\alpha + \beta) - 2\alpha\beta = 0$   
 $(\alpha + \beta) \{(\alpha + \beta) - 1\} = 0 \quad (\because \alpha\beta = 0)$   
 $\alpha + \beta = 0 \quad \dots (3)$   
 $\alpha + \beta = 1 \quad \dots (4)$   
 Solving (1) & (3)  
 $\alpha = 0, \beta = 0$   
 solving (1) & (4)  
 $\alpha(1 - \alpha) = 0 \Rightarrow \alpha = 0, 1$   
 $\Rightarrow \beta = 1, 0$   
 solving (2) & (4)

$$\alpha + \frac{1}{\alpha} = 1$$

$$\alpha^2 - \alpha + 1 = 0$$

$$(\alpha, \beta) \in (\omega, \omega^2)$$

Hence sample space  $\rightarrow (0, 0) (1, 1) (0, 1) (\omega, \omega^2)$

$$\therefore P(A) = \frac{2}{4} = \frac{1}{2}$$

6.  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc < 0$

P(E)  $\rightarrow$  when determinant value is negative

a	d	b	c
0	0	1	1
0	1	1	1
1	0	1	1

$$\therefore \text{Probability will be} \rightarrow 1 - \frac{3}{16} = \frac{13}{16}$$

7. Event (A)  $\rightarrow$  Plate has letter is palindrome  
 Event (B)  $\rightarrow$  Plate has digit is palindrome  
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{(26)^2 \times (10)^3}{(26 \times 10)^3} + \frac{(26)^3 \times (10)^2}{(26 \times 10)^3} - \frac{(26)^2 \times (10)^2}{(26 \times 10)^3}$$

9. When B rides A probability of winning of

$$A = \frac{2}{3} \times \frac{1}{6}$$

When C rides A probability of winning of

$$A = \frac{1}{3} \times \frac{1}{6} \times 3$$

$$\therefore P(A) = \frac{2}{18} + \frac{1}{6} = \frac{5}{18}$$

$$\text{Now odds against his winning} = \frac{18-5}{5} = \frac{13}{5}$$

15. Selection of 3 no. having 3 as minimum is  ${}^6C_2$   
 selection of 3 no. having 7 as maximum is  ${}^7C_2$   
 selection of 3 no. having 3 as minimum & 7 as maximum is  ${}^3C_1$ .

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= {}^6C_2 + {}^7C_2 - {}^3C_1$$

$$\text{sample space} = {}^{10}C_3$$

$$P(A) = \frac{{}^7C_2 + {}^6C_2 - {}^3C_1}{{}^{10}C_3}$$

17. Probability that 3 people out of 7 born on

$$\text{Wednesday} = \frac{{}^7C_3}{7^3}$$

Probability that 2 people out of remaining 4, born

$$\text{on Thursday is } \frac{{}^4C_2}{7^2}$$

$$\text{Probability of remaining 2 born on Sunday is } \frac{{}^2C_2}{7^2}$$

$$\therefore \text{required probability} = \frac{{}^7C_3}{7^3} \times \frac{{}^4C_2}{7^2} \times \frac{{}^2C_2}{7^2} = \frac{K}{7^6}$$

$$\Rightarrow K = 30$$

18. Events are defined as

$E_1$  = A rigged die is chosen

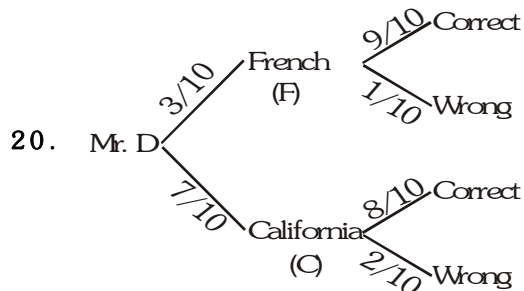
$E_2$  = A fair die is chosen

A = die shows 5 in all the three times

using Baye's Theorem :

$$P(E_1/A) = \frac{P(E_1) P(A/E_1)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)}$$

$$= \frac{\frac{1}{4} \times (1)^3}{\frac{1}{4} \times (1)^3 + \frac{3}{4} \times \left(\frac{1}{6}\right)^3} = \frac{216}{219}$$



$$P\left(\frac{C}{F}\right) = \frac{\frac{7}{10} \times \frac{2}{10}}{\frac{7}{10} \times \frac{2}{10} + \frac{3}{10} \times \frac{9}{10}} = \frac{14}{41}$$

21. After removing face cards & tens remaining cards  
 $= 52 - 16 = 36$

$$P(A) = \frac{4}{36}, P(H) = \frac{9}{36}, P(S) = \frac{9}{36}$$

$$P(A \cap S) = \frac{1}{36}$$

$$P(A \cap H) = \frac{1}{36}$$

$$P(A \cup S) = \frac{12}{36}$$

22. Ratio of their probabilities are 5 : 3 : 2

$$\Rightarrow 5x + 3x + 2x = 1 \Rightarrow x = \frac{1}{10}$$

their respective probabilities of A, B & C will be

$$= \frac{1}{2}, \frac{3}{10}, \frac{1}{5}$$

Now After A's accident

$$\frac{1}{2} - t = \frac{1}{3}$$

$$\Rightarrow t = \frac{1}{6} \text{ probability get reduced}$$

which will be shared between B & C

$$\Rightarrow 3y + 2y = \frac{1}{6} \Rightarrow y = \frac{1}{30}$$

$$\Rightarrow \text{Probability of C} = \frac{1}{5} + 2 \times \frac{1}{30} = \frac{4}{15}$$

$$\text{Probability of B} = \frac{3}{10} + 3 \times \frac{1}{30} = \frac{2}{5}$$

## EXERCISE - 02

## BRAIN TEASERS

1. Total cases of selecting two squares out of 64  $= {}^{64}C_2$   
 Favourable cases  $= 8.7 + 7.8$

$$\text{Probability} = \frac{8.7 + 7.8}{{}^{64}C_2} = \frac{1}{18}$$

3. Total cases  $= 15^7$

7 coupons are selected such that largest number on a selected coupon is 9. Fav. cases  $= 9^7 - 8^7$

$$\text{so required probability} = \frac{9^7 - 8^7}{15^7}$$

4. When dice are thrown

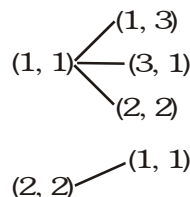
equal no.  $= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

Unequal no. whose sum is six  $=$

$$\{(1, 5), (5, 1), (4, 2), (2, 4)\}$$

$$\text{Probability of getting six with unequal no.} = \frac{4}{36}$$

When numbers are equal then getting sum six in four dice is



so probability of getting six  $=$

$$\frac{4}{36} + \frac{1}{6} \times \frac{3}{36} + \frac{1}{6} \times \frac{1}{36} = \frac{148}{1296}$$

7. a, b & c are three numbers

set (1, 2, 3, 4, 5)

sample space  $= 5^3$

$ab + c = \text{even}$

a	b	c	
odd	odd	odd	$= 3^3$
even	odd	even	$= 2 \quad 3 \quad 2$
odd	even	even	$= 2 \quad 3 \quad 2$
even	even	even	$= 2 \quad 2 \quad 2$

favourable case  $= 27 + 12 + 12 + 8 = 59$

$$\text{Probability to getting even no.} = \frac{59}{125}$$

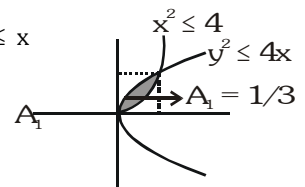
11.  $0 \leq x \leq 1, \quad 0 \leq y \leq 1$

Let A be the event  $y^2 \leq x$

B the event  $x^2 \leq y$

total area  $= 1$

$$P(A \cap B) = \frac{\text{Shaded area}}{\text{Total area}} = \frac{1}{3}$$



13. P(A) ; denote the probability that bus A will be late.  
 P(B) ; denote the probability that bus B will be late.

$$P(A) = \frac{1}{5}, P(B) = \frac{7}{25}$$

$$P\left(\frac{B}{A}\right) = \frac{9}{10} \Rightarrow P(A \cap B) = \frac{9}{50}$$

$$(i) P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$= 1 - \left(\frac{1}{5} + \frac{7}{25} - \frac{9}{50}\right) = \frac{35}{50} = \frac{7}{10}$$

$$(ii) \quad P\left(\frac{A}{B}\right) = \frac{P(A) \cdot P\left(\frac{B}{A}\right)}{P(B)} = \frac{\frac{1}{5} \times \frac{9}{10}}{\frac{7}{25}} = \frac{18}{28}$$

14.  $P(B) \neq 1$

$$(A) \quad P\left(\frac{A}{B^c}\right) = \frac{P(AB^c)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$$(B) \quad 1 \geq P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) \geq P(A) + P(B) - 1$$

$$(D) \quad P\left(\frac{A}{B^c}\right) + \frac{P(A^c)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{1 - P(B)} + \frac{P(A^c) - P(A^c \cap B)}{1 - P(B)} = \frac{P(A) - P(A \cap B) + 1 - P(A) - P(B) + P(A \cap B)}{1 - P(B)} = 1$$

15. The probability of event

$A \cap B$ ,  $A$ ,  $B$  &  $A \cup B$  are A.P.

$$P(B) - P(A) = P(A) - P(A \cap B) = P(A) \quad (\text{Given})$$

$$P(B) = 2P(A); \quad P(A \cap B) = 0$$

19.  $P(A)$  : denotes passing in A

$P(B)$  : denotes passing in B

$P(C)$  : denotes passing in C

$$P(A) = p, \quad P(B) = q, \quad P(C) = \frac{1}{2}$$

$$\text{probability that student is successful} = \frac{1}{2}$$

$$= p \cdot \frac{q}{2} + p \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

$$2pq + p = 2$$

Now check options

20. Total number of ways of selecting winners =  ${}^{16}C_8$

favourable ways =  ${}^2C_1 \cdot {}^{14}C_7$  (out of  $s_1$  &  $s_2$  & rest)

$$\text{so probability} = \frac{{}^2C_1 \cdot {}^{14}C_7}{{}^{16}C_8} = \frac{8}{15}$$

## EXERCISE - 03

## MISCELLANEOUS TYPE QUESTIONS

Fill in the blanks :

$$4. \quad 0 \leq \frac{1+3p}{3} \leq 1$$

$$0 \leq \frac{1-p}{4} \leq 1$$

$$0 \leq \frac{1-2p}{2} \leq 1$$

$$0 \leq \left(\frac{1+3p}{2}\right) + \left(\frac{1-p}{4}\right) + \left(\frac{1-2p}{2}\right) \leq 1$$

Match the column :

1. (B) Selecting two box out of 5 which remain empty =  ${}^5C_2$   
Favourable ways =  ${}^5C_2 (3^5 - 3(2^5 - 2) - 3)$   
Total ways =  $5^5$

$$\text{probability} = \frac{{}^5C_2 (3^5 - 3(2^5 - 2) - 3)}{5^5} = \frac{12}{25}$$

- (C) Let  $P(A)$  be the probability that the selected letters came from London  
 $P(B)$  be the probability that the selected letters came from clifton  
 $P(E)$  denotes the probability that ON is legible

$$P(A) = \frac{2}{5}, \quad P(B) = \frac{1}{6}$$

$$P\left(\frac{A}{E}\right) = \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(E)} = \frac{\frac{1}{2} \cdot \frac{2}{5}}{\frac{1}{2} \left(\frac{2}{5} + \frac{1}{6}\right)} = \frac{12}{17}$$

Assertion & reason :

1. Statement-I :

$$P\left(\frac{A \cap \bar{B}}{C}\right) = \frac{P((A \cap \bar{B}) \cap C)}{P(C)}$$

$$= \frac{P(A \cap C) - P((A \cap B) \cap C)}{P(C)}$$

Statement-II :

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$\{\because A \cap \bar{B} = A - (A \cap B)\}$$

Compression # 2

Let  $P(T_i)$  denotes probability of success if she studies is  $i$  hours.

$P(S)$  : Probability of success

$$P(T_{10}) = 0.8, \quad P(T_7) = 0.6, \quad P(T_4) = 0.4$$

$$1. \quad P(S) = 0.8 \cdot 0.1 + 0.2 \cdot 0.6 + 0.7 \cdot 0.4 = 0.48$$

$$2. \quad P\left(\frac{T_4}{S}\right) = \frac{P(T_4) \cdot P\left(\frac{S}{T_4}\right)}{P(S)} = \frac{0.4 \times 0.7}{0.48} = \frac{7}{12}$$

$$3. \quad P\left(\frac{T_4}{\bar{S}}\right) = \frac{P(T_4) \cdot P\left(\frac{\bar{S}}{T_4}\right)}{P(\bar{S})} = \frac{0.7 \times (1 - 0.4)}{0.52} = \frac{21}{26}$$



Also,

$\beta$  = probability of B getting the head on tossing secondly

$$= P(T_1 H_2 \text{ or } T_1 T_2 T_3 T_4 H_5 \text{ or } T_1 T_2 T_3 T_4 T_5 T_6 T_7 H_8 \text{ or } \dots)$$

$$= P(H)P(T) + P(H)P(T)^4 + P(H)P(T)^7 + \dots$$

$$= P(T) [P(H) + P(H)P(T)^3 + P(H)P(T)^6 + \dots]$$

$$= q \alpha = (1-p) \alpha = \frac{p(1-p)}{1-q^3}$$

Again we have  $\alpha + \beta + \gamma = 1$

$$\Rightarrow \gamma = 1 - (\alpha + \beta) = 1 - \frac{p + p(1-p)}{1-q^3}$$

$$= 1 - \frac{p + p(1-p)}{1-(1-p)^3} = \frac{1 - (1-p)^3 - p - p(1-p)}{1-(1-p)^3}$$

$$= \frac{1 - (1-p)^3 - 2p + p^2}{1-(1-p)^3} = \frac{p - 2p^2 + p^3}{1-(1-p)^3}$$

$$\text{Also, } \alpha = \frac{p}{1-(1-p)^3}, \beta = \frac{p(1-p)}{1-(1-p)^3}$$

## EXERCISE - 04[B]

## BRAIN STORMING SUBJECTIVE EXERCISE

1. Let  $a = 2^x$  &  $b = 2^y \Rightarrow \log_a b = \frac{y}{x}$  so  $\frac{y}{x}$  must be

integer. We know  $x, y \in \{1, 2, \dots, 25\}$

as  $y = nx$  so  $y$  is multiple of  $x$

If x	1	2	3	4	5	6	7	8	9	10 to 12	13 to 25
No. of possible values of y	24	11	7	5	4	3	2	2	1	1 each	no number

$$\text{required probability} = \frac{62}{25 \cdot 24} = \frac{31}{300}$$

$$\begin{aligned} 2. \quad P(6) &= P(24 \cup 42 \cup 33) \\ &= \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4} = \frac{2}{8} + \frac{1}{16} = \frac{5}{16} \\ P(666) &= \left(\frac{5}{16}\right)^3 \end{aligned}$$

$$A : \text{getting } 4 = \{2, 2\} \Rightarrow P(A) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\begin{aligned} P(\text{at least one } 4) &= 1 - P(\bar{A}\bar{A}\bar{A}) \\ &= 1 - \left(\frac{1}{4}\right)^3 = \frac{63}{64} \end{aligned}$$

$$3. \quad \frac{{}^{24}C_2}{{}^{64}C_2}$$

4.  $E_r$  : Scored exactly  $r$  points

$$\begin{aligned} P(E_n) &= P(E_{n-2}H \cup E_{n-1}T) \\ &= P(E_{n-2})P(H) + P(E_{n-1})P(T) \end{aligned}$$

$$P_n = P_{n-2} \cdot \frac{1}{2} + \frac{1}{2} P_{n-1}$$

$$P_n - P_{n-1} = \frac{1}{2} (P_{n-2} - P_{n-1})$$

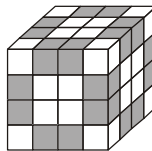
5. A : 1st man hit the target

B : 2nd man hit the target

C : 3rd man hit the target

$$(a) (i) P(A \cap B \cap C) = (0.3)(0.5)(0.4) = 0.6$$

$$(ii) P(\bar{A} \cap \bar{B} \cap \bar{C}) = (0.7)(0.5)(0.6) = 0.21$$



$$(b) (i) 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) = 0.79$$

$$\begin{aligned} (ii) P(A \cap \bar{B} \cap \bar{C}) &+ P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C) \\ &= (0.3)(0.5)(0.6) + (0.7)(0.5)(0.6) \\ &+ (0.7)(0.5)(0.4) \\ &= 0.09 + 0.21 + 0.14 = .44 \end{aligned}$$

(c) E : only one hits the target

$$= A\bar{B}\bar{C} \cup \bar{A}B\bar{C} \cup \bar{A}\bar{B}C$$

$$P(A\bar{B}\bar{C} / E) = \frac{(0.3)(0.5)(0.6)}{0.44} = \frac{0.09}{0.44}$$

$$6. \quad P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B) \quad \dots (1)$$

$$\text{also } \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = 0.1 \Rightarrow P(\bar{A} \cup B) = 0.02$$

$$P(A \cup B) = 0.98$$

$$P(A \cap B) = 0.4 + 0.8 - 0.98$$

$$= 0.22 \quad \dots (2)$$

Put (2) in (1)

$$\begin{aligned} P(\bar{A} \cup B) &= 0.6 + 0.8 - [P(B) - P(A \cap B)] \\ &= 0.6 + 0.8 - (0.8 - 0.22) = 0.82 \end{aligned}$$

$$(ii) P(\bar{A} \cup B) + P(A \cap \bar{B})$$

$$= P(A) + P(B) - 2P(A \cap B)$$

$$= 0.4 + 0.8 - 2(0.22) = 0.76$$

7. A : Target hit in 1st shot

B : Target hit in 2nd shot

C : Target hit in 3rd shot

$E_1$  : destroyed in exactly one shot

$E_2$  : destroyed in exactly two shot

$E_3$  : destroyed in exactly three shot

$$P(E_1) = P(E_1 \bar{A} \bar{B} \bar{C} \cup E_1 \bar{A} B \bar{C} \cup E_1 \bar{A} \bar{B} C)$$

$$= \frac{1}{3} \left[ \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} \right] = \frac{1+3+2}{3 \cdot 24} = \frac{1}{12}$$

$$P(E_2) = P(E_2 \bar{A} B \bar{C} \cup E_2 A \bar{B} \bar{C} \cup E_2 \bar{A} B C)$$

$$= \frac{7}{11} \left[ \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} \right] = \frac{7 \cdot 11}{11 \cdot 24} = \frac{7}{24}$$

$$P(E_3) = P(E_3ABC) = 1 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{4}$$

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) \\ = \frac{1}{12} + \frac{7}{24} + \frac{1}{4} = \frac{2+7+6}{24} = \frac{15}{24} = \frac{5}{8}$$

$$8. \quad P(\bar{H}/S) = \frac{P(\bar{H}S)}{P(S)} \quad \dots (1)$$

$$\text{also } \frac{P(HS)}{P(H)} = 1 - a \Rightarrow P(HS) = a(1 - a) \dots (2)$$

$$\text{also } \frac{P(\bar{S}\bar{H})}{P(\bar{H})} = 1 - a \Rightarrow P(\bar{S}\bar{H}) = (1 - a)^2$$

$$P(\bar{S} \cap \bar{H}) = P(\overline{S \cup H}) = (1 - a)^2 \\ \Rightarrow 1 - [P(S) + P(H) - P(SH)] = (1 - a)^2 \\ \Rightarrow [P(S) + a - a + a^2] = 2a - a^2 \\ P(S) = 2a(1 - a) \quad \dots (3)$$

$$\text{also } P(\bar{H}S) = P(S) - P(SH) \\ = 2a(1 - a) - a(1 - a) = a(1 - a) \quad \dots (4)$$

$$\text{from (3) \& (4) } P(\bar{H}/S) = \frac{1}{2}$$

9. A : Weather is favourable

$\bar{A}$  : Weather not good or low cloud

B : Reliability (instrument functions probability)

C : Safe landing

$$P(C/A) = p_1, \quad P(B) = P, \quad P(C/B) = p_2$$

$$P(C/\bar{B}) = p_2, \quad P(\bar{A}) = \frac{K}{100}$$

$$P(C) = P(A \cap C \cup \bar{A} \cap C \cup \bar{A} \cap \bar{C})$$

$$= \left(1 - \frac{K}{100}\right) p_1 + \frac{K}{100} [P p_1 + (1 - P)p_2]$$

$$P((\bar{A}BC \cup \bar{A}\bar{B}C)/C)$$

$$= \frac{\frac{K}{100} [P p_1 + (1 - P)p_2]}{\left(1 - \frac{K}{100}\right) p_1 + \frac{K}{100} [P p_1 + (1 - P)p_2]}$$

$$10. \quad \frac{{}^nC_1 (2^{n-1} - 1) + {}^nC_2 (2^{n-2} - 1) + \dots + {}^nC_{n-1} (2 - 1)}{(2^n - 1)^2}$$

$$= \frac{{}^nC_1 2^{n-1} + {}^nC_2 2^{n-2} + \dots + 2^n C_{n-1} - (2^n - 2)}{(2^n - 1)^2}$$

$$= \frac{(1 + 2)^n - 2^n - 1 - 2^n + 2}{(2^n - 1)^2} = \frac{3^n - 2^{n+1} + 1}{(2^n - 1)^2}$$

11. Total ways to ans. are  ${}^5C_1 + {}^5C_2 + \dots + {}^5C_5 = 31$   
If n chances are given then probability of success

$$\text{is } P(C_1 \cup C_2 \cup \dots \cup C_n) = \frac{1}{31} + \frac{1}{31} + \dots + \frac{1}{31}$$

$$= \frac{n}{31} \geq \frac{1}{3} \Rightarrow n \geq 10 \frac{1}{3}$$

$$\Rightarrow n = 11$$

12. C : Vehicle is Car  $P(\bar{C}) = 2/5$

$\bar{C}$  : Vehicle is Truck  $P(C) = 3/5$

E : Vehicle stops for fuel  $P(E/C) = 2/50$

$$P(\bar{E}/\bar{C}) = 1/30$$

$$P(E) = P(CE \cup \bar{C}E)$$

$$= P(CE) + P(\bar{C}E)$$

$$= \frac{2}{5} \left( \frac{2}{50} \right) + \frac{3}{5} \cdot \frac{1}{30} = \frac{1}{10} \left[ \frac{4}{25} + \frac{1}{5} \right] = \frac{9}{250}$$

$$P(C/E) = \frac{P(CE)}{P(E)} = \frac{4/250}{9/250} = \frac{4}{9}$$

Red Box		Green Box	
Red	Green	Green	Red
5	0	8	1
4	1	7	2
3	2	6	3
2	3	5	4
1	4	4	5
0	5	3	6

→ Non-prime

13.

→ Non-prime

$$\text{Required probability} = \frac{{}^6C_5 \cdot {}^8C_0 + {}^6C_1 \cdot {}^8C_4}{{}^{14}C_5} = \frac{213}{1001}$$

14.

$y^2 \backslash x^2$	0	1	4	5	6	9
0	✓					
1						✓
4					✓	
5				✓		
6			✓			
9		✓				

$$P(0) = \frac{1}{10} = P(5)$$

$$P(1) = P(4) = P(6)$$

$$= P(9) = \frac{2}{10}$$

$$P(00 + 19 + 91 + 55 + 64 + 46)$$

$$= \frac{1}{100} + \left( \frac{4}{100} \right) 4 + \frac{1}{100} = \frac{18}{100}$$

$$(b) \quad x^2 - y^2 = (x - y)(x + y)$$

$$1 \quad 4 \quad 7 \quad \dots \quad 3n - 2$$

$$2 \quad 5 \quad 8 \quad \dots \quad 3n - 1$$

$$3 \quad 6 \quad 9 \quad \dots \quad 3n$$

$$\text{Required probability} = \frac{{}^nC_2 + {}^nC_1 \cdot {}^nC_1 + {}^nC_2 + {}^nC_2}{{}^{3n}C_2}$$

$$= \frac{3n(n-1) + 2n^2}{3n(3n-1)} = \frac{5n-3}{9n-3}$$

15.  $E_r$  : hunters shoots the animal at r distance

$$P(E) = P(\text{escapes}) = 1 - P(\bar{E}_{2a} \bar{E}_{3a} \dots \bar{E}_{na})$$

$$= 1 - \left(1 - \frac{a^2}{4a^2}\right) \left(1 - \frac{a^2}{9a^2}\right) \dots \left(1 - \frac{a^2}{n^2 a^2}\right)$$

$$= 1 - \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{n^2}\right)$$

$$= 1 - \left(\frac{1.3}{2.2}\right) \left(\frac{2.4}{3.3}\right) \left(\frac{3.5}{4.4}\right) \dots \left(\frac{n-1}{n}\right)$$

$$= 1 - \frac{n+1}{2n} = \frac{n-1}{2n}$$

$$\text{odds against the hunter} = \frac{P(\bar{E})}{P(E)} = \frac{n+1}{n-1}$$

16.	Defective	Not Defective	Defective	Not Defective
	$n$	$N-n$	$m$	$M-m$

$$P(E_1) : \text{The selected article is from I}^{\text{st}} \text{ lot} = \frac{K}{K+L}$$

$$P(E_2) : \text{The selected article is from II}^{\text{nd}} \text{ lot} = \frac{L}{K+L}$$

$$\text{Required probability} = \frac{K}{K+L} \cdot \frac{n}{N} + \frac{L}{K+L} \cdot \frac{m}{M} =$$

$$\frac{KMn + LmN}{NM(K+L)}$$

17. A : A solves correctly      B : B solves correctly  
E : Commit same mistake  
F : same result

$$P(AB/F) = \frac{P(AB)}{P(AB) + P(\bar{A}\bar{B}E)} = \frac{\frac{1}{8} \cdot \frac{1}{12}}{\frac{1}{8} \cdot \frac{1}{12} + \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{1}{1001}}$$

$$= \frac{1001}{1078} = \frac{13}{14}$$

## EXERCISE - 05 [A]

## JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. Probability problem is not solved by A =  $1 - \frac{1}{2} = \frac{1}{2}$

$$\text{Probability problem is not solved by B} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{Probability problem is not solved by C} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\text{Probability of solving the problem} = 1 - P(\text{not solved by any body})$$

$$\therefore P = 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

2.  $P(A \cup B) = \frac{3}{4}$ ,  $P(A \cap B) = \frac{1}{4}$

$$P(\bar{A}) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$$

$$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\frac{1}{4} = \frac{1}{3} + P(B) - \frac{3}{4} \Rightarrow P(B) = \frac{2}{3}$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$$

3. Probability of getting odd  $p = \frac{3}{6} = \frac{1}{2}$

$$\text{Probability of getting others } q = \frac{3}{6} = \frac{1}{2}$$

$$\text{Variance} = npq = 5 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{4}$$

4. Out of 5 horses only one is the winning horse.  
The probability that Mr. A selected the losing horse
- $$= \frac{4}{5} \times \frac{3}{4}$$

$$\therefore \text{The probability that Mr. A selected the winning}$$

$$\text{horse} = 1 - \frac{4}{5} \times \frac{3}{4} = \frac{2}{5}$$

7.  $E = \{x \text{ is a prime number}\}$

$$P(E) = P(2) + P(3) + P(5) + P(7) = 0.62$$

$$F = \{x < 4\}, P(F) = P(1) + P(2) + P(3) = 0.50$$

$$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.62 + 0.50 - 0.35 = 0.77$$

8.  $\left. \begin{array}{l} np = 4 \\ npq = 2 \end{array} \right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$$P(X = 2) = {}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = 28 \cdot \frac{1}{2^8} = \frac{28}{256}$$

10. For a particular house being selected,

$$\text{Probability} = \frac{1}{3}$$

Probability (all the persons apply for the same

$$\text{house}) = \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) 3 = \frac{1}{9}.$$

14. Let A be the event that sum of digits is 8

**exhaustive cases**  $\rightarrow {}^{50}C_1$

favourable cases  $\rightarrow 08, 17, 26, 35, 44 = {}^5C_1$

$$P(A) = \frac{{}^5C_1}{{}^{50}C_1}$$

Let B be the event that product of digits is zero  
favourable cases  $\rightarrow$

$$\{00, 01, \dots, 09, 10, 20, 30, 40\} = {}^{14}C_1$$

$$\therefore P(B) = \frac{{}^{14}C_1}{{}^{50}C_1}$$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/{}^{50}C_1}{{}^{14}C_1/{}^{50}C_1} = \frac{1}{14}$$

15. The probability of at least one success

$$1 - \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$$

$$\left(\frac{3}{4}\right)^n \leq \frac{1}{10}$$

$$n \geq \log_{3/4} \left(\frac{1}{10}\right)$$

$$n \geq \frac{-\log 10}{\log_{10} 3 - \log_{10} 4}$$

$$n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$$

16. Required probability  $= \frac{{}^3C_1}{{}^9C_1} \cdot \frac{{}^4C_1}{{}^8C_1} \cdot \frac{{}^2C_1}{{}^7C_1} \cdot 3! = \frac{2}{7}$

17. Let terms of an AP

$$a, a + d, a + 2d, a + 3d$$

$$\therefore a \geq 1, a + 3d \leq 20$$

$$3d \leq 19 \Rightarrow d \leq \frac{19}{3}$$

so  $d = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$  and  $\pm 6$

statement 2 is wrong

if  $d = 1$

then  $a + 3d \leq 20$

$a \leq 17$

so 17 cases will

be there

Total case for  $d = \pm 1$  is 34

similarly  $d = -1$

so in this case also

17 cases will be there

$$18. P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$$

$$P(D) = \frac{P(C)}{P\left(\frac{C}{D}\right)} \leq 1$$

$$P(C) \leq P\left(\frac{C}{D}\right)$$

$$P\left(\frac{C}{D}\right) \geq P(C)$$

19. at least one failure = 1 - all success

$$1 \geq 1 - p^5 \geq \frac{31}{32}$$

$$0 \leq p^5 \leq \frac{1}{32}$$

$$0 \leq p \leq \frac{1}{2}$$

$$p \in \left[0, \frac{1}{2}\right]$$

20.  $P(A \cap B \cap C) = 0$

$$P\left(\frac{\bar{A} \cap \bar{B}}{C}\right) = \frac{P\{(\bar{A} \cap \bar{B}) \cap C\}}{P(C)} = \frac{P(\bar{A} \cap \bar{B})P(C)}{P(C)}$$

$$= \frac{[1 - P(A) - P(B) + P(A)P(B)]P(C)}{P(C)}$$

$$(\because P(A \cap B \cap C) = 0)$$

$$= \frac{P(C) - P(A)P(C) - P(B)P(C)}{P(C)}$$

$$= 1 - P(A) - P(B) = P(A^c) - P(B)$$

21. Let Events A denotes the getting min No. is 3 & B denotes the max. no. is 6

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$$

**Aliter :**

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{{}^4C_3 - (2)}{{}^8C_3}}{\frac{{}^6C_3 - {}^5C_3}{{}^8C_3}} = \frac{2}{10} = \frac{1}{5}$$

22.  $P(4\text{correct}) + P(5\text{ correct})$

$$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^5 = \frac{11}{3^5}$$



1.  $p_n$  denotes the probability that no two (or more) consecutive heads occur

$\Rightarrow p_n$  denotes the probability that 1 or no head occur. For  $n = 1$ ,  $p_1 = 1$  because in both cases we get less than two heads (H, T)

For  $n = 2$

$p_2 = 1 - p(\text{two head simultaneously occur})$

$$= 1 - p(HH) = 1 - pp = 1 - p^2$$

(probability of head is given as  $p$  not  $1/2$ )

For  $n \geq 3$ ,  $p_n = p_{n-1}(1-p) + p_{n-2}(1-p)p$

$$= (1-p)p_{n-1} + p(1-p)p_{n-2} \quad \text{Hence proved.}$$

2. (a) Let  $w_1 \rightarrow$  ball drawn in the first draw is white.

$b_1 \rightarrow$  ball drawn in the first draw is black.

$w_2 \rightarrow$  ball drawn in the second draw is white.

Then

$$P(w_2) = P(w_1).P(w_2/w_1) + P(b_1).P(w_2/b_1)$$

$$= \left(\frac{m}{m+n}\right)\left(\frac{m+k}{m+n+k}\right) + \left(\frac{n}{m+n}\right)\left(\frac{m}{m+n+k}\right)$$

$$= \frac{m(m+k) + mn}{(m+n)(m+n+k)}$$

$$= \frac{m(m+n+k)}{(m+n)(m+n+k)} = \frac{m}{m+n}$$

2. (b) Total number of favourable cases

$$= (3^n - 3 \cdot 2^n + 3) \cdot {}^6C_3$$

$\Rightarrow$  required probability

$$= \frac{(3^n - 3 \cdot 2^n + 3) \times {}^6C_3}{6^n}$$

5. (a) Here,  $P(A \cup B).P(A' \cap B')$

$$\Rightarrow \{P(A) + P(B) - P(A \cap B)\} \{P(A').P(B')\}$$

{Since A, B are independent  $\Rightarrow A', B'$  are independent}

$$\therefore P(A \cup B).P(A' \cap B')$$

$$\leq \{P(A) + P(B)\} \cdot \{P(A').P(B')\}$$

$$= P(A).P(A').P(B') + P(B).P(A').P(B') \quad \dots (1)$$

$$\leq P(A).P(B') + P(B).P(A')$$

{Since in (1),  $P(A') \leq 1$  and  $P(B') \leq 1$ }

$$\Rightarrow P(A \cup B).P(A' \cap B') \leq P(A).P(B') + P(B).P(A')$$

$$\Rightarrow P(A \cup B).P(A' \cap B') \leq P(C)$$

{as  $P(C) = P(A).P(B') + P(B).P(A')$ }

5. (b) Using Baye's theorem;  $P(B/A)$

$$\frac{\sum_{i=1}^3 P(A_i).P(B/A_i)}{\sum_{i=1}^3 P(A_i)}$$

where A be the event at least 4 white balls have been drawn.

$A_1$  be the event exactly 4 white balls have been drawn.  $A_2$  be the event exactly 5 white balls have been drawn.

$A_3$  be the event exactly 6 white balls have been drawn B be the event exactly 1 white ball is drawn from two draws.

$$\therefore P(B/A)$$

$$\frac{\frac{{}^{12}C_2 \cdot {}^6C_4 \cdot {}^{10}C_1 \cdot {}^2C_1}{{}^{18}C_6} + \frac{{}^{12}C_1 \cdot {}^6C_5 \cdot {}^{11}C_1 \cdot {}^1C_1}{{}^{18}C_6}}{\frac{{}^{12}C_2 \cdot {}^6C_4}{{}^{18}C_6} + \frac{{}^{12}C_1 \cdot {}^6C_5}{{}^{18}C_6} + \frac{{}^{12}C_0 \cdot {}^6C_6}{{}^{18}C_6}} = \frac{({}^{12}C_2 \cdot {}^6C_4 \cdot {}^{10}C_1 \cdot {}^2C_1) + ({}^{12}C_1 \cdot {}^6C_5 \cdot {}^{11}C_1 \cdot {}^1C_1)}{{}^{12}C_2 ({}^{12}C_2 \cdot {}^6C_4 + {}^{12}C_1 \cdot {}^6C_5 + {}^{12}C_0 \cdot {}^6C_6)}$$

5. (c) As three distinct numbers are to be selected from first 100 natural numbers

$$\Rightarrow n(S) = {}^{100}C_3$$

$E_{(\text{favourable events})}$  = All three of them are divisible by both 2 and 3.

$\Rightarrow$  divisible by 6 i.e., {6, 12, 18, ..., 96}

$$n(E) = {}^{16}C_3$$

$$P(E) = \frac{16 \times 15 \times 14}{100 \times 99 \times 98} = \frac{4}{1155}$$

10. Statement I : If  $P(H_i \cap E) = 0$  for some i, then

$$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$$

If  $P(H_i \cap E) \neq 0$  for  $\forall i = 1, 2, \dots, n$ , then

$$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$$

$$= \frac{P\left(\frac{E}{H_i}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right) P(H_i)$$

[as  $0 < P(E) < 1$ ]

Hence statement I may not always be true.

Statement II : Clearly,  $H_1 \cup H_2 \cup \dots \cup H_n = S$   
(sample space)

$$\Rightarrow P(H_1) + P(H_2) + \dots + P(H_n) = 1$$

12. Let B have n number of outcomes.

$$\text{so } P(B) = \frac{n}{10}, \quad P(A) = \frac{4}{10}$$

$$P(A \cap B) = \frac{4}{10} \cdot \frac{n}{10} = \frac{2n/5}{10}$$

$$\Rightarrow \frac{2n}{5} \text{ is an integer}$$

$$\Rightarrow n = 5 \text{ or } 10$$

17. C : Correct signal is transmitted

$\bar{C}$  : false signal is transmitted

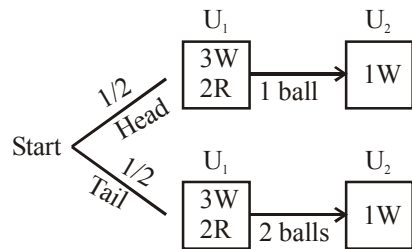
G : Original signal is green

R : Original signal is red

K : Signal received at station B is green.

$$\begin{aligned} P(G/K) &= \frac{P(G) \cdot P(K/G)}{P(K)} \\ &= \frac{P(GCC) + P(G\bar{C}\bar{C})}{P(GCC) + P(G\bar{C}\bar{C}) + P(RCC) + P(R\bar{C}\bar{C})} \\ &= \frac{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4}}{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{4} \times \frac{3}{4}} \\ &= \frac{40}{46} = \frac{20}{23} \end{aligned}$$

Paragraph for Question 18 and 19



18. Ans. (B)

Required probability

$$\begin{aligned} &= \frac{1}{2} \left( \frac{3}{5} \cdot 1 + \frac{2}{5} \cdot \frac{1}{2} \right) + \frac{1}{2} \left( \frac{{}^3C_2}{{}^5C_2} \cdot 1 + \frac{{}^2C_2}{{}^5C_2} \cdot \frac{1}{3} + \frac{{}^3C_1 {}^2C_1}{{}^5C_2} \cdot \frac{2}{3} \right) \\ &= \frac{1}{2} \left( \frac{4}{5} \right) + \frac{1}{2} \left( \frac{3}{10} + \frac{1}{30} + \frac{2}{5} \right) = \frac{2}{5} + \frac{11}{30} = \frac{23}{30} \end{aligned}$$

19. Ans. (D)

Required probability

$$= \frac{2/5}{2/5 + 11/30} \quad (\text{using Baye's theorem})$$

$$= \frac{12}{23}$$

$$21. P(X) = E_1 E_2 E_3 + E_1 E_2 \bar{E}_3 + E_1 \bar{E}_2 E_3 + \bar{E}_1 E_2 E_3$$

$$= \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$$

$$\Rightarrow P(X) = \frac{1}{4}$$

$$P\left(\frac{X_1^c}{X}\right) = \frac{P(X_1^c \cap X)}{P(X)} = \frac{1/32}{1/4} = \frac{1}{8}$$

P(Exactly two engines are functioning | x)

$$= \frac{7/32}{1/4} = \frac{7}{8}$$

$$P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)} = \frac{5/32}{1/4} = \frac{5}{8}$$

$$P\left(\frac{X}{X_1}\right) = \frac{P(X \cap X_1)}{P(X_1)} = \frac{7/32}{1/2} = \frac{7}{16}$$

$$22. 1 - \frac{{}^6C_1 \cdot 5^3}{6^4} = \frac{91}{216}$$

$$23. P(X \cap Y) = P(X), P(Y/X)$$

$$\Rightarrow P(X) = \frac{1}{2}$$

$$\text{Also } P(X \cap Y) = P(Y) \cdot P(X/Y)$$

$$\Rightarrow P(Y) = \frac{1}{3}$$

$$\Rightarrow P(X \cap Y) = P(X) \cdot P(Y)$$

$$\Rightarrow X, Y \text{ are independent}$$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$= \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$$

$$P(X^c \cap Y) = P(Y) - P(X \cap Y) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$$

$$\Rightarrow (A, B) \text{ are correct}$$

$$24. P(\text{Problem is solved by at least one of them})$$

$$= 1 - P(\text{solved by none})$$

$$= 1 - \left( \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8} \right)$$

$$= 1 - \frac{21}{256} = \frac{235}{256}$$

$$25. \text{Let } P(E_1) = p_1, P(E_2) = p_2, P(E_3) = p_3$$

$$\text{given that } p_1(1 - p_2)(1 - p_3) = \alpha \quad \dots\dots(i)$$

$$p_2(1 - p_1)(1 - p_3) = \beta \quad \dots\dots(ii)$$

$$p_3(1 - p_1)(1 - p_2) = \gamma \quad \dots\dots(iii)$$

$$\text{and } (1 - p_1)(1 - p_2)(1 - p_3) = p \quad \dots\dots(iv)$$

$$\Rightarrow \frac{p_1}{1-p_1} = \frac{\alpha}{p}, \frac{p_2}{1-p_2} = \frac{\beta}{p} \quad \& \quad \frac{p_3}{1-p_3} = \frac{\gamma}{p}$$

$$\text{Also } \beta = \frac{\alpha p}{\alpha + 2p} = \frac{3\gamma p}{p - 2\gamma}$$

$$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma$$

$$\Rightarrow \alpha p - 6p\gamma = 5\alpha\gamma$$

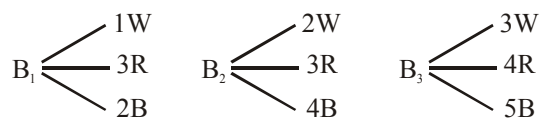
$$\Rightarrow \frac{p_1}{1-p_1} - \frac{6p_3}{1-p_3} = \frac{5p_1p_3}{(1-p_1)(1-p_3)}$$

$$\Rightarrow p_1 - 6p_3 = 0$$

$$\Rightarrow \frac{p_1}{p_3} = 6$$

**Paragraph for Question 26 to 27**

**26. Ans. (D)**



A = Total drawn balls are drawn & one is white, another is Red

$P(B_2|A)$  is to be determined

$P(B_2|A)$

$$= \frac{P(A|B_2)P(B_2)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)}$$

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P\left(\frac{A}{B_1}\right) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

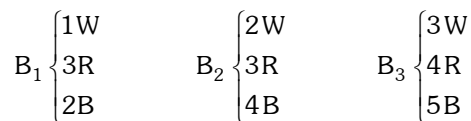
$$P(A|B_2) = \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2}$$

$$P(A|B_3) = \frac{{}^3C_1 \times {}^4C_1}{{}^9C_2}$$

By putting the values

$$P(B_2|A) = \frac{55}{181}$$

**27. Ans. (A)**



Probability of 3 drawn balls of same colour

$$= \frac{1}{6} \times \frac{2}{9} \times \frac{3}{12} + \frac{3}{6} \times \frac{3}{9} \times \frac{4}{12} + \frac{2}{6} \times \frac{4}{9} \times \frac{5}{12} = \frac{82}{648}$$