MATRIX

EXERCISE - 01 CHECK YOUR GRASP

2. $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow A_\alpha A_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = A_{\alpha+\beta}$

3. $60 = 2^2 \quad 3^1 \quad 5^1$

Number of divisor = 12

4. Hint : $x = 11 - y$ & $x + 5 = y$

6. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2+3 \\ 0 & 1 \end{bmatrix}$

on multiplying the matrix we get

$\begin{bmatrix} 1 & 1+2+.....+n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow n(n + 1) = 378 \quad 2 \Rightarrow n = 27$

8. $A^T = -A.$ & $A^TA = I$

$\Rightarrow A^2 = -I \Rightarrow A^{4n} = I$

$A^{4n-1} = A^{-1} \Rightarrow A^{4n-1} = A^T$ (A is orthogonal)

9. $AA^T = I \Rightarrow A^{-1} = A^T \Rightarrow A^T = \text{adj } A$

$(\because |A| = 1)$

$\therefore$ every element = cofactor

12. Hint : $B = A^{-1} \Rightarrow AB = I \Rightarrow 10AB = 10I$

13. Hint : $|2A^9 \; B^{-1}| = 2^2|A|^9 \dfrac{1}{|B|}$

14. Hint :Let $P = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}, Q = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$PAQ = R \Rightarrow A = P^{-1}RQ^{-1}$

19. $|A||\text{adj } A| = |A||A|^{n-1} = |A|^n = a^9$

28. $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - \lambda(a + d) + ad - bc = 0$

This is characteristic equation. Comparing with given equation we get

$k = ad - bc = |A|, \quad a + d = 0$

EXERCISE - 02 BRAIN TEASERS

1. $x = A^TBA$

$x^2 = A^TBA \,.\, A^TBA = A^TB^2A$

$x^{10} = A^TB^{10}A$

2. $A^2 = A \Rightarrow |A|^2 = |A| \Rightarrow |A| = 1$

$A(\text{adj } A) = |A| \; I$

$\text{adj}A = A^{-1}$

also $A^2 = A$

$A = I \Rightarrow \text{adj } A = I$

$(\text{adj } A)^2 = I \Rightarrow (\text{adj } A)^2 = \text{adj } A$

3. $|A| = x(yz - 8) - 3(z - 8) + 2 (2 - 2y)$

$= 60 - 20 + 28 = 68$

$A \; (\text{adj}A) = |A| I = \begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$

4. $BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$S = t_r(A) + t_r\left(\dfrac{A}{2}\right) + t_r\left(\dfrac{A}{4}\right) + \ldots \infty$

$S = 3 + \dfrac{3}{2} + \dfrac{3}{4} + \ldots \infty = 3 \; \dfrac{1}{1-\dfrac{1}{2}} = 6$

5. $A^T = BCD$

$AA^T = ABCD \Rightarrow AA^T = S \Rightarrow AA^T = S^T$

$\Rightarrow S = S^T$

$D^T \; C^T \; B^T \; A^T = ABC \,.\, DAB \,.\, CDA.\; BCD$

$(ABCD)^T = (ABCD) \; (ABCD) \; (ABCD)$

$S^T = S^3 \Rightarrow S = S^3 \Rightarrow S^2 = S^4$

6. $|A^TA^{-1}| = |A^T| \; |A^{-1}| = |A^T| \dfrac{1}{|A|} = 1$

$\Rightarrow f(x) = 1$

14. Hint : $\Delta_1 = \Delta_0^2, \; \Delta_2 = \Delta_1^2 = \Delta_0^{2^2}$

$\therefore \quad \Delta_n = \Delta_0^{2n}$

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Assertion & Reason :

1. x, y, z are not all zero

$\Rightarrow$ system has infinite solution.

$\Delta = 0$

$$\Rightarrow \Delta = -\frac{(a+b+c)}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$$

but a, b, c are distinct $\Rightarrow a + b + c = 0$

Statement-I is false & Statement-II is true.

4. Statement-I :

$$\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}$$

$$\begin{bmatrix}a+2c & b+2d\\ -a-c & -b-d\end{bmatrix}=\begin{bmatrix}a-b & 2a-b\\ c-d & 2c-d\end{bmatrix}$$

$\Rightarrow$ $2c = -b$ & $b = a - d$

$\therefore$ infinite matrix are there.

Statement-II :

$AI = IA \Rightarrow A(adjA) = (adjA)A$

$\Rightarrow AA^{-1} = A^{-1}A$

5. St-I : $x=\underset{\substack{\downarrow\\ P}}{\begin{bmatrix}\cos\pi/12 & \sin\pi/12\\ -\sin\pi/12 & \cos\pi/12\end{bmatrix}}\ A\ \underset{\substack{\downarrow\\ P^T}}{\begin{bmatrix}\cos\pi/12 & -\sin\pi/12\\ \sin\pi/12 & \cos\pi/12\end{bmatrix}}$

$\Rightarrow PP^T = I$

Now $x = PAP^T$

$\Rightarrow x^2 = PAP^TPAP^T \quad \Rightarrow x^2 = PA^2P^T$

$\Rightarrow x^2 = PAP^T \quad \Rightarrow x^2 = x$

St-II : $Q = PAP^T \quad \Rightarrow Q^2 = PAP^T\ .\ PAP^T$

If A is idempotent then $Q^2 = PA^2P^T$

$\Rightarrow Q^n = PA^nP^T$

Comprehension # 3 :

Hint : $|A_0| = 0$

$B_1 = B_2 = B_3 = \ldots\ldots = B_{49} = B_0$

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

2. $\ell = 1 + 2 + 3 + \ldots\ldots + n \Rightarrow \ell = \frac{n(n+1)}{2}+1$;

$m = n + 1 \quad p = \frac{n(n-1)}{2}$

3. $$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$$

$$\begin{bmatrix}a+2c & b+2d\\ 3a+4c & 3b+4d\end{bmatrix}=\begin{bmatrix}a+3b & 2a+4b\\ c+3d & 2c+4d\end{bmatrix}$$

On equating we get

$2c = 3b$ ... (1)

$2d = 2a + 3b$ ... (2)

$c = d - a$ ... (3)

$$\frac{d-b}{a+c-b} = \frac{d-b}{d-b} = 1$$

Let $d = \alpha$, $c = \beta$ then $A = \begin{bmatrix}\alpha-\beta & 2\beta/3\\ \beta & \alpha\end{bmatrix}$

4. $n(A) = 4$; $n(B) = 2$; $n(C) = 4$; $n(D) = 1$; $|D| = 18$

$$\frac{n(C)(|D|^2+n(D))}{n(A)-n(B)}=\frac{4(18^2+1)}{2} = 650$$

5. $A = \begin{bmatrix}0 & 1\\ 3 & 0\end{bmatrix}$; $A^2 = \begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} = 3I$

$$(1 + 3 + 3^2 + 3^3 + 3^4)\ I\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}0\\ 11\end{bmatrix}$$

$$(121)\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}0\\ 11\end{bmatrix} \Rightarrow \begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}0\\ 1/11\end{bmatrix}$$

8. $A^2 = I \quad \Rightarrow x = 2, 4, 6, \ldots\ldots$

$$\sum(\cos^x\theta+\sin^x\theta)$$

$$= (\cos^2\theta+\sin^2\theta)+(\cos^4\theta+\sin^4\theta)+\ldots\ldots$$

$$= (\cot^2\theta+\tan^2\theta) \geq 2$$

15. (a) $$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 9\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 4\\ 6\end{bmatrix}$$

$AX = B$

$|A| = 2$

$$X = A^{-1}B = \frac{(adjA)}{|A|}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{\begin{bmatrix}6 & -5 & 1\\ -6 & 8 & -2\\ 2 & -3 & 1\end{bmatrix}\begin{bmatrix}3\\ 4\\ 6\end{bmatrix}}{2} = \begin{bmatrix}2\\ 1\\ 0\end{bmatrix}$$

EXERCISE - 04 [B] CONCEPTUAL SUBJECTIVE EXERCISE

2. $|B| = |adj\ A| = |A|^2 = 9$

$\frac{S}{2} = \frac{a/b}{1 - a/b^2} = \frac{ab}{b^2 - a} = \frac{27}{81-3} = \frac{27}{78} = \frac{9}{26}$

$(ab^2 + a^2b + 1)\ S = 225$

3.

a_{11}	a_{21}	+	a_{12}	a_{22}	=	0	
1	1		1	–1	→	4	ways
–1	–1		–1	1	→	4	ways

4. $B = (I - A)(I + A)^{-1}$

$B^T = [(I + A)^{-1}]^T\ (I - A)^T = [(I + A)^T]^{-1}\ (I - A)^T$

$= (I + A^T)^{-1}\ (I - A^T) = (I - A)^{-1}(I + A)$

$BB^T = (I - A)(I + A)^{-1}(I - A)^{-1}(I + A)$

$= (I - A)\{(I - A)(I + A)\}^{-1}\ (I + A)$

$= (I - A)(I - A)^{-1}(I + A)^{-1}(I - A) = I$

8. $\left[\underset{I}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} + \underset{A}{\begin{bmatrix} 0 & 2 & a \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}}\right]^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow\ (I + A)^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow\ I + nA + \frac{n(n-1)}{2}A^2 = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$

(A^3, A^4....... is a null matrix)

on solving it we get

$n = 9\ \ \&\ \ a = 191$

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

3. $A = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$

$10\ B = \begin{vmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{vmatrix}$

$B = A^{-1}$

$AB = AA^{-1} = I$

$10\ AB = 10I$

$(A)\ (10B) = 10I$

$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

$\begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

$5 - \alpha = 0$

$\boxed{\alpha = 5}$

4. $A^2 - A + I = 0$

multiplying by A^{-1}

$A^{-1}\ AA - A^{-1}\ A + A^{-1}\ I = 0$

$IA - I + A^{-1} = 0$

$\boxed{A^{-1} = I - A}$

7. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ $\qquad a, b \in N$

$AB = \begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$

$BA = \begin{pmatrix} a & 2b \\ 3b & 4b \end{pmatrix}$

For AB = BA

b = a → their are infinite

Natural number for which a = 6

so Infinite matrix B possible

8. $|A^2| = 25$

$|A|^2 = 25$

$|A| = \pm 5$

$\begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix} = \pm 5$

$25\alpha = \pm 5$

$\alpha = \pm \frac{1}{5}$

9. $A^2 = I$

$|A^2| = |I|$

$|A^2| = 1$

$|A| = \pm 1$

statement–1 :

If $A \neq I, A \neq -I$

but $|A| = \pm 1$

so this statement is true

statement–2 :

Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$|A| = -1$ $\quad$ $tr(A) = 0$

but $A \neq I, A \neq -I$

so statement-2 is false

14. $A^T = A$

$B^T = B$

St-1 :

$(A(BA))^T = (BA)^T A^T$

$= A^T B^T A^T = A(BA) \rightarrow$ symetric

$((AB)A))^T = A^T B^T A^T = (AB) A \rightarrow$ symetric

Statement - 1 is true

St-2 :

$(AB)^T = B^T A^T = BA$

if AB = BA then

$(AB)^T = BA = AB$

St.- 2 is true

but Not a correct expalnation.

15. **St–1 :** The value of det. of skew sym. matrix of odd order is always zero. So St-I. is true.

St–II : This st. is not always true depends on the order of matrix.

$|-A| = -|A|$ if order is odd, so St-–II is wrong.

St-I is true and St-II is false.

16. Since H is a diagonal matrix. We know that product of two diagonal matrix is always a diagonal matrix.

So $H^{70} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \cdots \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$ 70 times

$= \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$

17. $(P^2 + Q^2) P = P^3 + Q^2 P$ ... (1)

$(P^2 + Q^2) Q = P^2 Q + Q^3$ ... (2)

Equation (1) – Equation (2)

$(P^2 + Q^2)(P - Q) = P^3 - Q^3 + Q^2 P - P^2 Q$

$(P^2 + Q^2)(P - Q) = 0 \quad \because (P \neq Q)$

$P^2 + Q^2 = 0$

so $|P^2 + Q^2| = 0$

18. $A^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$

and $A^{-1} A U_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ ...(1)

$A^{-1} A U_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ ...(2)

Eq. (1) + (2)

$U_1 + U_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

3. $|M - I| = |M - M M^T|$

$|M - I| = |M| \ |I - M|$

$\Rightarrow |M - I| = |I - M|$

$\Rightarrow |M - I| = (-1)^3 |M - I|$

$\Rightarrow |M - I| = 0$

4. $AX = U$

$$\Rightarrow \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$$

has infinitely many solutions.

$\Rightarrow |A| = 0 \Rightarrow (c - d)(ab - 1) = 0$

& (adj A) U = 0

$$\begin{bmatrix} bc-bd & -c & d \\ d-c & ac & -ad \\ 0 & 1-ab & ab-1 \end{bmatrix} \begin{bmatrix} f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} fbc-fbd-gc+dh \\ fd-fc+agc-adh \\ g-abg+adh-h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow fd - fc + agc - agh = 0$ ... (1)

$$BX = V \Rightarrow \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

$|B| = a(dh - gc) + fc - fd = 0$ (from (1))

$\therefore$ system can't have unique solution

Now X = (adj B)V

$$= \begin{bmatrix} dh-gc & g-h & c-d \\ fc & ah-f & -ac \\ -fd & -g+af & ad \end{bmatrix} \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

if $afd \neq 0 \Rightarrow$ (adj B) $V \neq 0$

$\therefore$ $adfd \neq 0$ then BX = V is inconsislent

5. $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \Rightarrow |A| = 6$$

$$A^{-1} \Rightarrow \frac{adjA}{|A|} = \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}[A^2 + cA + dI]$$

$$\frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \frac{1}{6}\left\{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix} + \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}\right\}$$

on comparing we get

$-1 = 5 + c \Rightarrow c = -6$

$1 = 14 + 4c + d \Rightarrow 1 = 14 - 24 + d$

$d = 11$

6. $PP^T = I$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \text{ \& so on}$$

$Q = PAP^T$

$Q^2 = (PAP^T)(PAP^T) = PA^2P^T$

$Q^{2005} = PA^{2005}P^T$

$$x = P^T(PA^{2005}P^T)P \Rightarrow x = A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

10. (c) (i) If A is symmetric, $A^T = A$

$$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} a & c \\ b & a \end{bmatrix}$$

$\Rightarrow b = c$

If A is skew symmetric, $A^T = -A$

$$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} -a & -c \\ -b & -a \end{bmatrix}$$

$\Rightarrow a = 0, b + c = 0$

$\because$ $b, c \geq 0 \Rightarrow a = 0, b = 0, c = 0$

Now, det(A) = $a^2 - bc$

$= a^2 - b^2$ ($\because$ b = cfor A being symmetric or skew symmetric or both)

$= (a - b)(a + b)$ is divisible by p.

Let $(a - b)(a + b) = \lambda p$, $\lambda \in I$

Range of (a + b) is 0 to 2p − 2 which includes only one multiple of p i.e. p

$\therefore$ a + b = p & a − b $\in$ I

$\Rightarrow$ possible number of pairs of a & b will be p − 1.

Also, range of (a − b) is 1 − p to p − 1 which includes only one multiple of p i.e. 0

$\therefore$ a − b = 0 & a + b $\in$ I

$\Rightarrow$ Possible number of pairs of a & b will be p.

Hence total number of A in T_p will be

p + p − 1 = 2p − 1

(iii) Total number of A in $T_P = p^3$

when a ≠ 0 & det(A) is divisible by p, then number of A will be $(p-1)^2$

When a = 0 & det(A) is divisible by p, then number of A will be 2p−1.

So, total number of A for which det(A) is divisible by p

$= (p-1)^2 + 2p - 1$

$= p^2$

So number of A for which det(A) is not divisible by p

$= p^3 - p^2$

11. *(Comment : Although 3 3 skew symmetric matrices can never be non-singular. Therefore the information given in question is wrong. Now if we consider only non singular skew symmetric matrices M & N, then the solution is-)*

Given $M^T = -M$

$N^T = -N$

$MN = NM$

according to question $M^2N^2(M^TN)^{-1}(MN^{-1})^T$

$= M^2N^2N^{-1}(M^T)^{-1}(N^{-1})^TM^T$

$= M^2N^2N^{-1}(-M)^{-1}(N^T)^{-1}(-M)$

$$\left[\begin{array}{l} MN = NM \\ (MN)^{-1} = (NM)^{-1} \\ N^{-1}M^{-1} = M^{-1}N^{-1} \end{array}\right.$$

$= -M^2 N M^{-1} N^{-1} M$

$= -M^2 N N^{-1} M^{-1} M = -M^2$

12. $\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix}$

$=1 - c\omega - a(\omega - \omega^2 c) = (1 - c\omega) - a\omega(1 - c\omega) = (1 - c\omega)(1 - a\omega)$

for non singular matrix

$$c \neq \frac{1}{\omega} \ \& \ a \neq \frac{1}{\omega}$$

$\Rightarrow$ $c \neq \omega^2, \quad a \neq \omega^2$

$\Rightarrow$ a & c must be ω & b can be ω or ω^2

$\therefore$ total matrices = 2

14. $|Q| = \begin{vmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{vmatrix}$

$\Rightarrow |Q| = 2^2.2^3.2^4.\begin{vmatrix} a_{11} & 2a_{12} & 2^2a_{13} \\ a_{21} & 2a_{22} & 2^2a_{23} \\ a_{31} & 2a_{32} & 2^2a_{33} \end{vmatrix}$

$= 2^2.2^3.2^4|P|.2^3$

$= 2^2.2^3.2^4.2.2^3 = 2^{13}$

15. $P^T = 2P + I$

$\Rightarrow P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I$

$\Rightarrow P = 4P + 3I$

$\Rightarrow P = -I$

$\Rightarrow PX = -X$

16. $|adjP| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$

$\Rightarrow |P|^2 = 4 \Rightarrow |P| = \pm 2$