INVERSE TRIGONOMETRIC FUNCTION

EXERCISE - 01 — CHECK YOUR GRASP

3. $\sec\left[\sin^{-1}\left(-\sin\frac{50\pi}{9}\right)+\cos^{-1}\cos\left(-\frac{31\pi}{9}\right)\right]$

$= \sec\left[-\sin^{-1}\left(\sin\frac{50\pi}{9}\right)+\cos^{-1}\cos\left(\frac{31\pi}{9}\right)\right]$

$= \sec\left[-\sin^{-1}\left(-\sin\frac{4\pi}{9}\right)+\cos^{-1}\left(-\cos\frac{4\pi}{9}\right)\right]$

$= \sec\left[\frac{4\pi}{9}+\pi-\frac{4\pi}{9}\right] = \sec\pi = -1.$

5. $(\sin^{-1}x+\sin^{-1}y)^2 = \pi^2$

$\Rightarrow \sin^{-1}x+\sin^{-1}y = \pm\pi$

$\Rightarrow \sin^{-1}x = \sin^{-1}y = \frac{\pi}{2}$

or $\sin^{-1}x = \sin^{-1}y = -\frac{\pi}{2}$

$\Rightarrow x^2+y^2 = 2.$

8. $x = \frac{2\pi}{3}+\frac{\pi}{4}+\frac{\pi}{3} = \frac{5\pi}{4}$

$y = \cos\left(\frac{1}{2}\sin^{-1}\left(\sin\frac{5\pi}{8}\right)\right)$

$= \cos\left(\frac{1}{2}\left(\pi-\frac{5\pi}{8}\right)\right) = \cos\frac{3\pi}{16}.$

10. $\tan^{-1}2+\tan^{-1}3 = \operatorname{cosec}^{-1}x$

$\Rightarrow \pi+\tan^{-1}(-1) = \operatorname{cosec}^{-1}x$

$\Rightarrow \pi-\frac{\pi}{4} = \operatorname{cosec}^{-1}x \Rightarrow \frac{3\pi}{4} = \operatorname{cosec}^{-1}x$

$\Rightarrow$ no solution. $\left\{-\frac{\pi}{2}\le\operatorname{cosec}^{-1}x\le\frac{\pi}{2}\right\}$

15. Hint : $y = \cos^{-1}\frac{x^2}{1+x}$

Now $-1\le\frac{x^2}{1+x}\le 1$

19. $(\tan^{-1}x)^2 - 3\tan^{-1}x + 2 \ge 0$

$(\tan^{-1}x-1)(\tan^{-1}x-2) \ge 0$

we know that $\tan^{-1}x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

so $\tan^{-1}x \ge 2$ (not possible) or $\tan^{-1}x \le 1$

$\Rightarrow x \in (-\infty, \tan 1]$

20. $\tan^2(\sin^{-1}x) > 1$

either $\tan(\sin^{-1}x) > 1 \Rightarrow \sin^{-1}x > \tan^{-1}1$

$\Rightarrow \sin^{-1}x > \sin^{-1}\frac{1}{\sqrt{2}}$

$\Rightarrow x > \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} < x < 1$

or

$\tan(\sin^{-1}x) < -1 \Rightarrow \sin^{-1}x < \tan^{-1}(-1)$

$\Rightarrow \sin^{-1}x < \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) \Rightarrow -1 < x < -\frac{1}{\sqrt{2}}$

so $x \in (-1, 1) - \left[\frac{-\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right]$

22. $\cos\left[\frac{1}{2}\cos^{-1}\cos\left(2\pi+\frac{4\pi}{5}\right)\right]$

$= \cos\left(\frac{1}{2}\times\frac{4\pi}{5}\right) = \cos\frac{2\pi}{5}$

$= \cos\left(\pi-\frac{3\pi}{5}\right) = -\cos\left(\frac{3\pi}{5}\right)$

& $\cos\frac{2\pi}{5} = \sin\left(\frac{\pi}{2}-\frac{2\pi}{5}\right) = \sin\left(\frac{\pi}{10}\right)$

EXERCISE - 02 — BRAIN TEASERS

2. Let $\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right) = \theta$

expression $= \sin\theta+\cos\theta = \sqrt{2}(\sin(\theta+\pi/4))$

$= \sqrt{2}\sin\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)+\cot^{-1}(1)\right)$

$= \sqrt{2}\sin\left(\frac{\pi}{2}-\tan^{-1}1+\frac{\pi}{2}-\frac{1}{2}\cot^{-1}\frac{3}{4}\right)$

$= \sqrt{2}\sin\left(\pi-\tan^{-1}(1)-\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

Also $\cot 2\theta = \frac{-3}{4}$

so $\sin\theta+\cos\theta = \sqrt{1+\sin 2\theta} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{2}}{\sqrt{10}}$

$\left\{\text{As } \cot^{-1}-\left(\frac{3}{4}\right)\in(0,\pi),\ \sin^{-1}2\theta \text{ will be positive}\right\}$

7. $2.\cos^{-1}x = \cot^{-1}\left(\frac{2x^2-1}{2|x|\sqrt{1-x^2}}\right)$

Let $\cos^{-1}x = \theta$

$2\theta = \cot^{-1}\left(\frac{\cos 2\theta}{2|\cos\theta|\sin\theta}\right)$

Case I : If $\cos\theta > 0$, $x > 0 \Rightarrow 0 < x < 1$ then

$2\theta = \cot^{-1}\cot 2\theta = 2\theta$ (identity)

Case II : $\cos\theta < 0$, which not satisfy the equation

8. $\sin^{-1}\sqrt{1-x^2} + \frac{\pi}{2} = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

Let $\theta = \sin^{-1}x,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, $x \neq 0$, $\theta \neq 0$

so $\sin^{-1}\cos\theta + \frac{\pi}{2} = \cot^{-1}\cot\theta$

$\sin^{-1}\sin\left(\frac{\pi}{2}-\theta\right) + \frac{\pi}{2} = \cot^{-1}\cot\theta$

Case I : If $0 < \theta \le \frac{\pi}{2}$

then $0 < \frac{\pi}{2} - \theta < \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - \theta + \frac{\pi}{2} = \theta \Rightarrow \theta = \frac{\pi}{2}$

$\sin\theta = 1 = x$

Case II : If $-\frac{\pi}{2} \le \theta < 0$

$\Rightarrow 0 < -\theta \le \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \le \frac{\pi}{2} - \theta < \pi$

then $\pi - \left(\frac{\pi}{2} - \theta\right) + \frac{\pi}{2} = \pi + \theta$

$\pi + \theta = \pi + \theta$

$-\frac{\pi}{2} \le \theta < 0 \Rightarrow -1 \le \sin\theta < 0 \Rightarrow -1 \le x < 0$

12. $\because$ $\alpha > 0 \Rightarrow (\alpha, \sin^{-1}\alpha)$ lies in 1^{st} quandrant

$\Rightarrow$ $k > 0$

Also the extreme point on the graph of $y = \sin^{-1}x$ is $\left(1, \frac{\pi}{2}\right)$

$\Rightarrow 1 + \frac{\pi}{2} - k < 0 \Rightarrow k > 1 + \frac{\pi}{2}$

$\Rightarrow k \in \left(1 + \frac{\pi}{2}, \infty\right)$

13. Let $y = \frac{2(x^2+1)+1}{(x^2+1)} = 2 + \frac{1}{(x^2+1)}$

$\Rightarrow 2 < y \le 3$

Now $\sin^{-1}\sin y \le \pi - \frac{5}{2} \Rightarrow \pi - y \le \pi - \frac{5}{2}$

$\Rightarrow y \ge \frac{5}{2} \Rightarrow \frac{2x^2+3}{x^2+1} \ge \frac{5}{2}$

Now it can be solved

14. $< a_n >$ is $1, 2, 2^2, \ldots\ldots\ldots 2^{n-1}$

$< b_n >$ is $1, \frac{1}{2}, \frac{1}{2^2}, \ldots\ldots \frac{1}{2^{n-1}}$

$t_r = \cot^{-1}(2a_r + b_r) = \tan^{-1}\left(\frac{1}{2.2^{r-1} + \frac{1}{2^{r-1}}}\right)$

$= \tan^{-1}\left(\frac{2^{r-1}}{2.2^{(r-1)}.2^{(r-1)} + 1}\right)$

$= \tan^{-1}\left(\frac{2.2^{r-1} - 2^{r-1}}{1 + 2.2^{r-1}.2^{r-1}}\right) = \tan^{-1}2.2^{r-1} - \tan^{-1}2^{r-1}$

Now $\lim_{n\to\infty}\sum_{r=1}^{n} t_r = (\tan^{-1}2 - \tan^{-1}1)$

$+ (\tan^{-1}2.2 - \tan^{-1}2) + \ldots\ldots + (\tan^{-1}2.2^{n-1} - \tan 2^{n-1})$

$= \tan^{-1}\infty - \tan^{-1}1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

15. $\sum_{r=1}^{\infty} T_r = \cot^{-1}\left(r^2 + \frac{3}{4}\right) = \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{1 + r^2 - \frac{1}{4}}\right)$

$= \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + \left(r - \frac{1}{2}\right)\left(r + \frac{1}{2}\right)}\right)$

$= \sum_{r=1}^{\infty}\tan^{-1}\left(r + \frac{1}{2}\right) - \sum_{r=1}^{\infty}\tan^{-1}\left(r - \frac{1}{2}\right)$

Now it can be solved.

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Assersion & Reason :

2. $x(x-2)(3x-7) = 2$

$\Rightarrow 3x^3 - 13x^2 + 14x - 2 = 0$

$s_1 = r + s + t = \frac{13}{3}$; $s_2 = \frac{14}{3}$, $s_3 = \frac{2}{3}$

$\tan^{-1}r + \tan^{-1}s + \tan^{-1}t = \pi + \tan^{-1}\left[\frac{s_1 - s_3}{1 - s_2}\right]$

$= \pi + \tan^{-1}[-1] = \frac{3\pi}{4}$

Hence statement-I and statement-II both are true.

Comprehension # 1 :

1. (i) $\sin\left(\frac{\cos^{-1}x}{y}\right) = 1$

$\Rightarrow \frac{\cos^{-1}x}{y} = 2n\pi + \frac{\pi}{2}$ & $y \neq 0$

$\Rightarrow \cos^{-1}x = (4n+1)\frac{\pi}{2}y$

when $n = 0 \Rightarrow \cos^{-1}x = \frac{\pi}{2}y$

when $y = 1$, $x = 0$ $\{0 < \frac{\pi}{2}y \leq \pi$

$y = 2$, $x = -1$ $\Rightarrow 0 < y \leq 2\}$

when

$n = 1$ or > 1 $\cos^{-1}x = \frac{5\pi}{2}$ y or more(reject)

$n = -1$ or < -1 $\cos^{-1}x = \frac{-3\pi}{2}$ y or more(reject)

(ii) $\cos\left(\frac{\sin^{-1}x}{y}\right) = 0$

$\Rightarrow \frac{\sin^{-1}x}{y} = (2n+1)\frac{\pi}{2}$ & $y \neq 0$

$n = 0$ $\sin^{-1}x = \frac{\pi}{2}y$

$\{\frac{-\pi}{2} \leq \frac{\pi}{2}y \leq \frac{\pi}{2} \Rightarrow -1 \leq y \leq 1\}$

When $y = 1$, $x = 1$
$y = -1$, $x = -1$

$n = -1$ $\sin^{-1}x = -\frac{\pi}{2}y$

When $y = 1$, $x = -1$
$y = -1$, $x = 1$

Other values of n & y are out of range.

1. (0, 1) & (– 1, 2)
2. (1, 1), (1, –1), (–1, 1), (–1, –1)
3. one one onto

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

2 (b) $y = \sec^{-1}(\log_3 \tan x + \log_{\tan x} 3)$

$\Rightarrow \tan x > 0$ & $\tan x \neq 1$

so $D = (2n\pi, 2n\pi + \frac{\pi}{2}) \cup ((2n+1)\pi, 2n\pi + \frac{3\pi}{2})$

$- \{x \mid x = 2n\pi + \frac{\pi}{4}$ or $2n\pi + \frac{5\pi}{4}\}$, $n \in I$

4. $f(x) = \cos^{-1}x + \cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right)$

(a) $f\left(\frac{2}{3}\right) = \cos^{-1}\frac{2}{3} + \cos^{-1}\frac{1}{2} - \cos^{-1}\frac{2}{3} = \frac{\pi}{3}$

(b) $f\left(\frac{1}{3}\right) = \cos^{-1}\frac{1}{3} - \cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{3} = 2\cos^{-1}\frac{1}{3} - \frac{\pi}{3}$

10. Let $\tan x = a$ & $\tan y = b$

$\sin^2 x + \sin^2 y < 1 \Rightarrow \frac{a^2}{1+a^2} + \frac{b^2}{1+b^2} < 1$

Solving, we get $a^2b^2 < 1 \Rightarrow |ab| < 1$

$\Rightarrow \sin^{-1}(ab) \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

13. $\alpha = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$

Put $x = \tan\theta$

$0 < \theta < \frac{\pi}{4}$ $\{\because 0 < x < 1\}$

$\alpha = 2\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = 2\tan^{-1}\tan\left(\frac{\pi}{4} + \theta\right)$

$= \frac{\pi}{2} + 2\theta$

$\beta = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x = \tan\theta$

$= \sin^{-1}(\cos 2\theta) = \frac{\pi}{2} - \cos^{-1}\cos(2\theta)$

$\Rightarrow 0 < \theta < \frac{\pi}{4}$ $\{\because 0 < x < 1\}$

$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$

$\therefore \beta = \frac{\pi}{2} - 2\theta \Rightarrow \alpha + \beta = \pi$

Similarly value of $\alpha+\beta$ can be found out for $x > 1$

14. (d) $\sin^{-1} x - \sin^{-1}(3x - 2) = \sin^{-1}\sqrt{1-x^2}$

$\Rightarrow \quad x\sqrt{1-(3x-2)^2} - \sqrt{1-x^2}\,(3x - 2)$

$= \sqrt{1-x^2}$

$\Rightarrow \quad x^2(-9x^2 + 12x - 3) = (1 - x^2)(9x^2 - 6x + 1)$

$\Rightarrow \quad 6x^3 - 11x^2 + 6x - 1 = 0$

$\Rightarrow \quad x = 1, \dfrac{1}{2}, \dfrac{1}{3}$ (reject)

(e) $\cos(\sin^{-1} x + \sin^{-1}(1 - x)) = \cos\cos^{-1} x$

$\sqrt{1-x^2}\ \sqrt{1-(1-x)^2} - x(1 - x) = x$

$(2x - x^2)(2x - 1) = 0 \qquad (0 \le x \le 1)$

$x = 0, \dfrac{1}{2}, 2$

but $x = 2$ is not possible

(f) $2\tan^{-1}x\left(\dfrac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\dfrac{2a}{1-a^2}\right) - \tan^{-1}\left(\dfrac{2b}{1-b^2}\right)$

$\Rightarrow \quad 2\tan^{-1} x = 2\tan^{-1} a - 2\tan^{-1} b$

$\Rightarrow \quad \tan^{-1}x = \tan^{-1}\dfrac{a-b}{1+ab} \Rightarrow x = \dfrac{a-b}{1+ab}$

15. (c) $\tan^{-1}\left(\dfrac{(x+1)-x}{1+x(x+1)}\right) + \tan^{-1}\left(\dfrac{(x+2)-(x+1)}{1+(x+2)(x+1)}\right) + \ldots$

$\ldots + \tan^{-1}\left(\dfrac{(x+n)-(x+n-1)}{1+(x+n)(x+n-1)}\right)$

$= \tan^{-1}(x + 1) - \tan^{-1} x + \tan^{-1}(x + 2)$

$-\tan^{-1}(x+1) + \ldots + \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$

$= \tan^{-1}(x + n) - \tan^{-1} x$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. (a) $x^2 - 5x + 13 > 0 \Rightarrow x \in R$

$1 - \log_7(x^2 - 5x + 13) > 0$

$\Rightarrow \quad x^2 - 5x + 13 < 7$

$\Rightarrow \quad 2 < x < 3$

Also $-1 \le \dfrac{3}{2 + \sin\frac{9\pi}{2}x} \le 1$

$\Rightarrow \quad -2 - \sin\frac{9\pi}{2}x \le 3 \le 2 + \sin\frac{9\pi}{2}x$

$\Rightarrow \quad \sin\frac{9\pi}{2}x \ge -5 \quad \& \quad 1 \le \sin\dfrac{9\pi}{2}x$

$\Rightarrow \quad 1 = \sin\dfrac{9\pi}{2}x$

$\Rightarrow \quad x \in R \quad \& \quad x = \dfrac{4n+1}{9}, n \in I$

$\therefore \quad D_f = \left\{\dfrac{21}{9}, \dfrac{25}{9}\right\}$

(b) $\sqrt{\sin(\cos x)}$

$\Rightarrow \quad 2n\pi - \dfrac{\pi}{2} \le x \le \dfrac{\pi}{2} + 2n\pi$

Now $-2\cos^2 x + 3\cos x + 1 > 0$

$\Rightarrow \quad \dfrac{3-\sqrt{17}}{4} < \cos x < \dfrac{3+\sqrt{17}}{4}$

Also $-1 \le \dfrac{2\sin x + 1}{2\sqrt{2\sin x}} \le 1 \qquad (\sin x > 0)$

$\Rightarrow \quad (2\sin x + 1 - 2\sqrt{2\sin x}) \le 0$

$\& \quad 2\sin x + 1 + 2\sqrt{2\sin x} \ge 0$

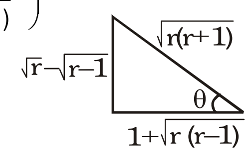
$\Rightarrow \quad (\sqrt{2\sin x} - 1)^2 \le 0$

$\& \quad (\sqrt{2\sin x} + 1)^2 \ge 0 \Rightarrow \sin x = \dfrac{1}{2}$

$\Rightarrow \quad x = 2n\pi + \dfrac{\pi}{6}, \sin x \ge 0 \quad \& \quad x \in R$

$\Rightarrow \quad x = 2n\pi + \dfrac{\pi}{6}$

3. Hint : $T_r = \sin^{-1}\left(\dfrac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right)$



let $\sin\theta = \dfrac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}$

$T_r = \tan^{-1}\left(\dfrac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r}\sqrt{r-1}}\right)$

$T_r = \tan^{-1}(\sqrt{r}) - \tan^{-1}(\sqrt{r-1})$

Now proceed.

5. $\cos^{-1} x + (\sin^{-1} y)^2 = \dfrac{K\pi^2}{4}$

$\Rightarrow \quad 0 \le \cos^{-1} x < \pi \quad \& \quad 0 \le (\sin^{-1} y)^2 \le \dfrac{\pi^2}{4}$

$\Rightarrow \quad 0 \le \cos^{-1} x + (\sin^{-1} y)^2 \le \pi + \dfrac{\pi^2}{4}$

$\Rightarrow \quad 0 \le K\dfrac{\pi^2}{4} \le \pi + \dfrac{\pi^2}{4}$

$\Rightarrow \quad 0 \le K \le \dfrac{4}{\pi} + 1$

Also $(\sin^{-1} y)^2 \cos^{-1} x = \dfrac{\pi^4}{16}$

let $\cos^{-1} x = t$ then

$$t\left(\frac{K\pi^2}{4} - t\right) = \frac{\pi^4}{16}$$

$\Rightarrow \quad t^2 - \dfrac{K\pi^2}{4} t + \dfrac{\pi^4}{16} = 0$

For solution $D \geq 0 \Rightarrow \dfrac{K^2\pi^4}{16} - \dfrac{\pi^4}{4} \geq 0$

$\Rightarrow \quad K^2 \geq 4$

$\therefore$ only integral value of $K = 2$

7. (a) $(\cot^{-1} x)^2 - 5 \cot^{-1} x + 6 > 0$

$(\cot^{-1} x - 3)(\cot^{-1} x - 2) > 0$

$\Rightarrow \quad \cot^{-1}x < 2$ or $\cot^{-1}x > 3$

as $\cot^{-1} x$ is decreasing function

$\cot^{-1} x < 2$ or $\cot^{-1} x > 3$

$\Rightarrow \quad x > \cot 2$ or $x < \cot 3$

9. $f(x) = \cot^{-1} (x^2 + 4x + \alpha^2 - \alpha)$

$\Rightarrow \quad x^2 + 4x + \alpha^2 - \alpha \geq 0$

Now $f(x)$ has to be onto

$\Rightarrow \quad D = 0$

$\Rightarrow \quad 16 - 4(\alpha^2 - \alpha) = 0$

$\Rightarrow \quad \alpha = \dfrac{1 \pm \sqrt{17}}{2}$

11. $f(x)=(\sin^{-1}x+\cos^{-1}x)^3 - 3\sin^{-1}x \cos^{-1}x(\sin^{-1}x + \cos^{-1}x)$

$$= \left(\frac{\pi}{2}\right)^3 - \frac{3\pi}{2} \cos^{-1} x\left(\frac{\pi}{2} - \cos^{-1} x\right)$$

$$= \left(\frac{\pi}{2}\right)^3 + \frac{3\pi}{2} \cos^{-1} x\left(\cos^{-1} x - \frac{\pi}{2}\right)$$

$$= \left(\frac{\pi}{2}\right)^3 + \frac{3\pi}{2} \left\{\left(\cos^{-1} x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right\}$$

$$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\cos^{-1} x - \frac{\pi}{4}\right)^2$$

$$f_{\min}(x) = \frac{\pi^3}{32}$$

$$f_{\max}(x) = \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\pi - \frac{\pi}{4}\right)^2 = \frac{7\pi^3}{8}$$

12. (b) $\sin(\sin^{-1}(\log_{1/2}x)) + 2\,|\cos(\sin^{-1}(x/2 - 1))| = 0$

$-1 \leq \log_{1/2} x \leq 1 \Rightarrow \dfrac{1}{2} \leq x \leq 2$ ....(i)

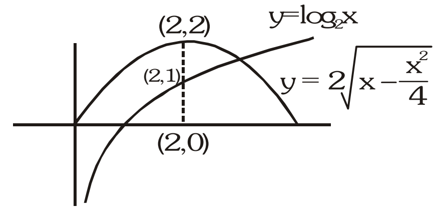
and $-1 \leq \dfrac{x}{2} - 1 \leq 1$

$\Rightarrow \quad 0 \leq \dfrac{x}{2} \leq 2 \Rightarrow 0 \leq x \leq 4$ ....(ii)

From (i) & (ii), $\dfrac{1}{2} \leq x \leq 2$

Also $\log_{1/2} x + 2\left|\sqrt{x - \dfrac{x^2}{4}}\right| = 0$

$2\sqrt{x - \dfrac{x^2}{4}} = \log_2 x$... (1)



From graph it is clear that equation (1) does not have any solution in $\left[\dfrac{1}{2}, 2\right]$

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

2. $\sin^{-1}x = 2\sin^{-1}a$

$$-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2} \leq 2\sin^{-1}a \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \leq \sin^{-1}a \leq \frac{\pi}{4}$$

$$-\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$$

3. $\cos^{-1}\left[\dfrac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\dfrac{y^2}{4}}\right] = \alpha$

$$\left(\frac{xy}{2} - \cos\alpha\right)^2 = (1 - x^2)\left(1 - \frac{y^2}{4}\right)$$

$$\frac{x^2y^2}{4} - \frac{2xy}{2}\cos^2\alpha + \cos^2\alpha = 1 - x^2 - \frac{y^2}{4} + \frac{x^2y^2}{4}$$

$$x^2 + \frac{y^2}{4} - xy\cos^2\alpha = \sin^2\alpha$$

$$4x^2 + y^2 - 4xy\cos^2\alpha = 4\sin^2\alpha$$

6. $2y = x + z$

and $2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$\Rightarrow \quad \dfrac{2y}{1-y^2} = \dfrac{x+z}{1-xz}$

$\Rightarrow \quad y^2 = zx$

$\Rightarrow \quad x = y = z$

2. For $f(x)=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$ to be defined and real

$\sin^{-1}2x+\pi/6\ge 0$

$\Rightarrow \quad \sin^{-1}2x \ge -\frac{\pi}{6}$ ....(1)

But we know that $-\pi/2\le\sin^{-1}2x\le\pi/2$ ....(2)

Combining (1) and (2), $-\pi/6\le\sin^{-1}2x\le\pi/2$

$\Rightarrow \sin(-\pi/6)\le 2x\le\sin(\pi/2)\Rightarrow -1/2\le 2x\le 1$

$\Rightarrow \quad -1/4\le x\le 1/2$

$\therefore \quad D_f=\left[-\frac{1}{4},\frac{1}{2}\right]$

6. $\sqrt{1+x^2}\quad\left[\left\{x\cos\left(\cot^{-1}x\right)+\sin\left(\cot^{-1}x\right)\right\}^2-1\right]^{1/2}$

Let $x=\cot\theta$

then $\operatorname{cosec}\theta\left[\left\{\cot\theta\cos\theta+\sin\theta\right\}^2-1\right]^{1/2}$

$=\operatorname{cosec}\theta\ \left[\operatorname{cosec}^2\theta-1\right]^{1/2}$

$=\sqrt{1+\cot^2\theta}\ \cot\theta = x\sqrt{1+x^2}$

7. $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$

$=\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\frac{2n(n+1)}{2}\right)\right)$

$=\cot\left(\sum_{n=1}^{23}\tan^{-1}\frac{1}{1+n(n+1)}\right)$

$=\cot\left(\sum_{n=1}^{23}\tan^{-1}\frac{(n+1)-n}{1+n(n+1)}\right)$

$=\cot\left(\sum_{n=1}^{23}\left(\tan^{-1}(n+1)-\tan^{-1}n\right)\right)$

$=\cot\left(\tan^{-1}24-\tan^{-1}1\right)$

$=\cot\left(\tan^{-1}\frac{23}{25}\right)=\cot\left(\cot^{-1}\left(\frac{25}{23}\right)\right)=\frac{25}{23}$

8. (P) $$\left(\frac{1}{y^2}\left(\frac{\frac{1}{\sqrt{1+y^2}}+\frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}\right)^2+y^4\right)^{\frac{1}{2}}$$

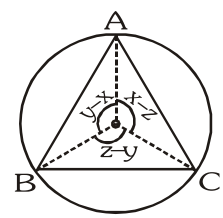
$$\Rightarrow \left(\left(\frac{\left(1+y^2\right)y^2\left(1-y^2\right)}{y^2}\right)+y^4\right)^{\frac{1}{2}}$$

$$\Rightarrow \left(\left(1-y^4\right)+y^4\right)^{\frac{1}{2}}=1$$

(Q) A(cosx, sinx), B(cosy, siny) & C(cosz, sinz) lie on circle $x^2+y^2=1$

$\because$ (0,0) is circumcentre as well as centroid of ΔABC

$\Rightarrow \quad \Delta ABC$ is an equilateral triangle



$$y-x=\frac{2\pi}{3}$$

$$\cos\frac{y-x}{2}=\cos\frac{\pi}{3}=\frac{1}{2}$$

(R) $\cos 2x\left(\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}-x\right)\right)$

$=\sin 2x\left(1-\tan x\right)$

$\sqrt{2}\sin x\cos 2x=\sin 2x\left(1-\tan x\right)$

$\sin x\left(\sqrt{2}\cos 2x-2\left(\sin x-\cos x\right)\right)=0$

$\Rightarrow \sin x=0$ or $\sin x=\cos x$ or $\sin x+\cos x=\sqrt{2}$

$\Rightarrow \quad \sec x=\pm 1$ or $\sec x=\pm\sqrt{2}$

(S) $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right)$

$\frac{|x|}{\sqrt{1-x^2}}=\frac{\sqrt{6}x}{\sqrt{1+6x^2}}$ (x > 0)

$\Rightarrow \quad 1-6x^2=6+6x^2$

$\Rightarrow \quad x=\sqrt{\frac{5}{12}}=\frac{\sqrt{5}}{2\sqrt{3}}$