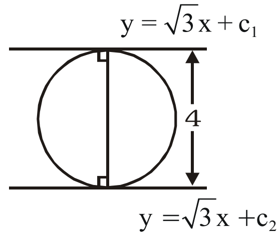
EXERCISE - 01 CHECK YOUR GRASP

7. Distance between both lines is diameter of the circle

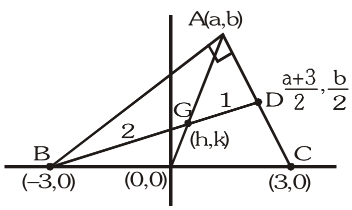


$$4=\left|\frac{c_1-c_2}{\sqrt{1+3}}\right|$$

$|c_1 - c_2| = 8$

8. If three lines are given such that no two of them are parallel and they are not concurrent then a definite triangle is formed by them. There are four circles which touch sides of a triangle (3-excircles and 1-incircle).

9.



$$\angle BAC = 90° \Rightarrow \left(\frac{b}{a+3}\right)\left(\frac{b}{a-3}\right)=-1$$

$\Rightarrow$ $b^2 = -(a^2 - 9)$ $\Rightarrow$ $a^2 + b^2 = 9$ ....(i)

Now BG : GD = 2 : 1

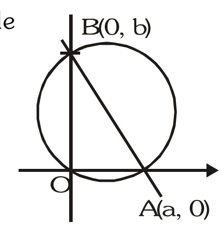
$$\Rightarrow 3h=\frac{2(a+3)}{2}+1\times-3 \quad \Rightarrow \quad a = 3h$$

$$\& \quad 3k=2\left(\frac{b}{2}\right)+1\times 0 \quad \Rightarrow \quad b = 3k$$

substitute value of a & b in equation (i)

$9h^2 + 9k^2 = 9 \Rightarrow x^2 + y^2 = 1$

10. Let centroid of the triangle OAB be (α, β)



$\therefore$ $a = 3\alpha,\ b = 3\beta$

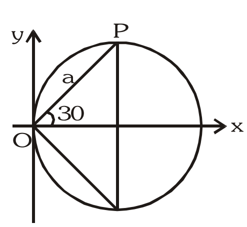
$a^2 + b^2 = 36k^2$

$\Rightarrow$ $9\alpha^2 + 9\beta^2 = 36k^2$

$\therefore$ Locus of (α,β) is

$x^2 + y^2 = 4k^2$

11. Coordinates of point P will be (acos30 , asin30) P lies on the circle,



$\Rightarrow$ $a^2\cos^2 30 + a^2\sin^2 30$

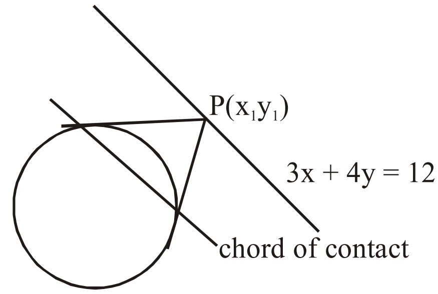
$= 2a\cos 30$

$\Rightarrow$ $a^2 = 2a\cos 30$

$\Rightarrow$ $a=\sqrt{3}$

$$\text{Area}=\frac{\sqrt{3}a^2}{4}=\frac{3\sqrt{3}}{4}$$

24. Let $P(x_1, y_1)$ be a point on the line $3x + 4y = 12$. Equation of variable chord of contact of $P(x_1, y_1)$ wrt circle $x^2 + y^2 = 4$ is



$xx_1 + yy_1 - 4 = 0$ .....(1)

Also $3x_1 + 4y_1 - 12 = 0$

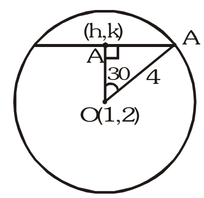
$x_1 + \frac{4}{3}y_1 - 4 = 0$ ......(2)

Comparing (1) & (2)

$$x = 1,\ y=\frac{4}{3}$$

$\therefore$ variable chord of contact always passes through $(1, \frac{4}{3})$

25.



In ΔOAB

$$\cos 30=\frac{\sqrt{(h-1)^2+(k-2)^2}}{4}=\frac{\sqrt{3}}{2}$$

Squaring both sides, we get the desired locus.

26. Centre of circles lie on the perpendicular bisector of the given line.

$$\Rightarrow \frac{k-3}{h-2}=\frac{2}{5}$$

locus of P(h, k) is $2x - 5y + 11 = 0$

28. $y^2 - 2xy + 4x - 2y = 0$

$y(y - 2x) -2(y - 2x) = 0$

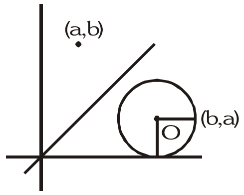
$\Rightarrow$ $y = 2$ and $y = 2x$ are the normals.

Now point of intersection of normals will give the centre of the circle i.e. (1, 2)

Radius of circle will be $\sqrt{2}$

$\therefore$ equation of circle : $(x - 1)^2 + (y - 2)^2 = 2$

29. Reflection of point (a, b) on the line $y = x$ will be (b, a)



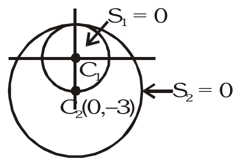
$(x - b)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2bx - 2ay + b^2 = 0.$

32. $S_1 : x^2 + y^2 = 9 \Rightarrow C_1(0, 0), \quad r_1 = 3$

$S_2 : x^2 + y^2 + 6y + c = 0$

$\Rightarrow C_2(0, -3), r_2 = \sqrt{9-c}$



Now, $C_1C_2 = r_2 - r_1$

$3 = \sqrt{9-c} - 3$

$36 = 9 - c \qquad \Rightarrow c = -27$

33. $C_1C_2 = r_1 \pm r_2$

$\Rightarrow (g_1 - g_2)^2 + (f_1 - f_2)^2 = \left(\sqrt{g_1^2 + f_1^2} \pm \sqrt{g_2^2 + f_2^2}\right)^2$

$\Rightarrow -2g_1g_2 - 2f_1f_2 = \pm 2 \sqrt{g_1^2 + f_1^2} \cdot \sqrt{g_2^2 + f_2^2}$

$\Rightarrow g_1f_2 - g_2f_1 = 0$

$\Rightarrow \dfrac{g_1}{g_2} = \dfrac{f_1}{f_2}$

35. Let the centre of circle be $(-g, -f)$

Using condition of orthogonality :

$2(g_1g_2 + f_1f_2) = C_1 + C_2$

$2(2g - 3f) = 9 + C \qquad$ ....(i)

$2\left(-\dfrac{5g}{2} + 2f\right) = -2 + C \qquad$ ....(ii)

Subtract (ii) from (i)

$2\left[\dfrac{9g}{2} - 5f\right] = 11 \qquad \Rightarrow \quad 9g - 10f = 11$

replacing $(-g)$ by h & $(-f)$ by k.

$-9h + 10k = 11$

$\Rightarrow \quad 9x - 10y + 11 = 0$

EXERCISE - 02 BRAIN TEASERS

1. Let equation of the circle be

$x^2 + y^2 + 2gx + 2fy + \lambda = 0$

$\left(t, \dfrac{1}{t}\right)$ be a point on the circle

$\therefore \; t^2 + \dfrac{1}{t^2} + 2gt + 2f\dfrac{1}{t} + \lambda = 0$

$t^4 + 2gt^3 + \lambda t^2 + 2ft + 1 = 0$

roots of the above equation are a, b, c, & d

$\therefore$ abcd = 1

4. Consider $\quad a = \cos\theta, \quad b = \sin\theta$

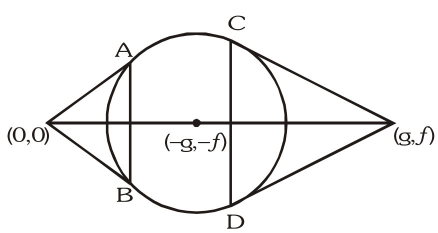
$\qquad m = \cos\phi, \quad n = \sin\phi$

Now, $am \pm bn = \cos\theta\cos\phi \pm \sin\theta\sin\phi$

$am \pm bn = \cos(\theta \mp \phi)$

$\therefore |am \pm bn| \le 1$

6.



Equation of AB : $\quad gx + fy + c = 0 \quad$ ....(i)

Equation of CD : $\quad gx + fy + g(x+g) + f(y+f) + c = 0$

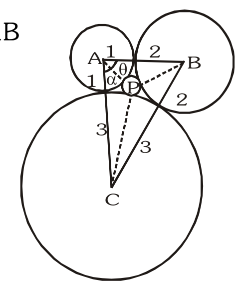
$gx + fy + \dfrac{g^2 + f^2 + c}{2} = 0 \quad$ .....(ii)

Distance between AB & CD will be

$$\left|\frac{\dfrac{g^2 + f^2 - c}{2}}{\sqrt{g^2 + f^2}}\right| = \frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$$

9. Δ ABC is right angle

Applying cosine rule in ΔPAB



$\cos\theta = \dfrac{3^2 + (1+r)^2 - (2+r)^2}{2.3(1+r)}$

$= \dfrac{3-r}{3(1+r)}$

Again applying cosine rule in ΔPAC

$\cos\alpha = \dfrac{(1+r)^2 + 4^2 - (3+r)^2}{2.4(1+r)} = \dfrac{2-r}{2(1+r)}$

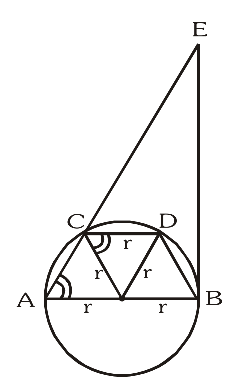
$\because \alpha + \theta = 90$

$\alpha = 90 - \theta \qquad \Rightarrow \cos\alpha = \sin\theta$

$\left(\dfrac{3-r}{3(r+1)}\right)^2 + \left(\dfrac{2-r}{2(r+1)}\right)^2 = 1$

12. $\angle CAB = 60°$

In ΔABE



$\cos 60° = \dfrac{AB}{AE}$

$\Rightarrow \quad AE = 2AB$

Solving above equation and get value of r.

14. Equation of variable circle which touch the x-axis at origin is $x^2 + y^2 + \lambda y = 0$

Let the pole of the above circle be P(h, k)

Equation of polar is

$$hx + ky + \frac{\lambda}{2}(y + k) = 0$$

$$hx + \left(k + \frac{\lambda}{2}\right)y + \frac{\lambda k}{2} = 0 \qquad \dots (1)$$

and the equation of given polar is

$\ell x + my + n = 0$ ... (2)

comparing (1) and (2)

$$\frac{h}{\ell} = \frac{k + \frac{\lambda}{2}}{m} = \frac{\lambda k}{2n}$$

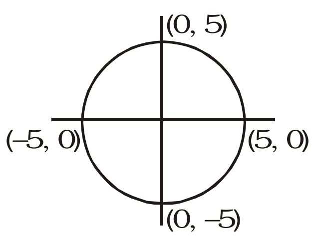
$$\Rightarrow mh = \ell k + \frac{\ell\lambda}{2} \text{ and } nh = \frac{\ell\lambda k}{2}$$

$$\Rightarrow mh = \ell k + \frac{nh}{k} \Rightarrow mhk = \ell k^2 + nh$$

$\therefore x(my - n) - \ell y^2 = 0$

16. $x^2 + y^2 < 25$

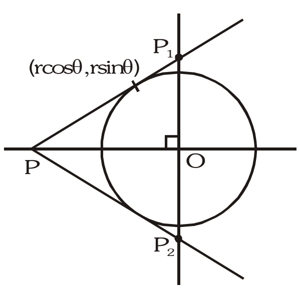
Number of integral coordinate satisfying above inequality in first quadrant is 13 i.e. (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2),



$\therefore$ Total number of integral coordinates are

$$13 \quad 4 + \underbrace{4 \times 4}_{\text{on coordinate axes}} + \underbrace{1}_{\text{origin}} = 69$$

19.



Were $r = 5\sqrt{2}$

Equation of PP_1 : $x\cos\theta + y\sin\theta = r$

point P will be : $(r\sec\theta, 0)$

point P_1 will be : $(0, r\,\mathrm{cosec}\theta)$

Area of $\Delta\, PP_1P_2$ will be $\left(\frac{1}{2} \times r\sec\theta \times r\,\mathrm{cosec}\,\theta\right) \times 2$

$$\Delta PP_1P_2 = \frac{2r^2}{\sin 2\theta}$$

Area of ΔPP_1P_2 will be minimum if $\sin 2\theta = 1$ or -1.

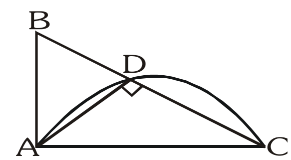
$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{4}, \ \theta = \frac{3\pi}{4}$$

$$\Rightarrow \quad P : \quad (5\sqrt{2} \times \sqrt{2}, 0) \text{ or } (5\sqrt{2}(-\sqrt{2}), 0)$$

(10, 0) or (–10, 0)

22. Triangles BAC and BDA are similar

$$\therefore \frac{AC}{AD} = \frac{BC}{AB}$$



$$AC = \frac{BC.AD}{AB}$$

$\{AB^2 = BD\ .\ BC\}$

$$= \frac{AB.AD}{BD} = \frac{AB.AD}{\sqrt{AB^2 - AD^2}}$$

23. Let the equation of the circle is -

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(1)

which touches the line $\ell x + my + n = 0$

$$\therefore \left|\frac{-\ell g - mf + n}{\sqrt{\ell^2 + m^2}}\right| = \sqrt{g^2 + f^2 - c} \qquad \dots (2)$$

and circle (1) is orthogonal to the circle $x^2 + y^2 = 9$

$\therefore 0 \quad g + 0 \quad f = c - 9$

$\Rightarrow c = 9$ ... (3)

from (2) & (3)

$$\left|\frac{-\ell g - mf + n}{\sqrt{\ell^2 + m^2}}\right| = \sqrt{g^2 + f^2 - 9}$$

$\therefore$ locus of (– g, – f) is

$(\ell x + my + n)^2 = (x^2 + y^2 - 9)(\ell^2 + m^2)$

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Assertion & Reason :

1. $x^2 + y^2 + 2x + 2y - 2 = 0$

$(x + 1)^2 + (y + 1)^2 = 4$

Director circle of the above circle is -

$(x + 1)^2 + (y + 1)^2 = 8$

$x^2 + y^2 + 2x + 2y - 6 = 0$

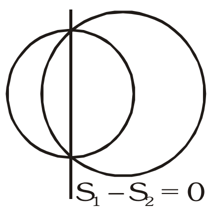
$\therefore$ Tangents drawn from any point on the second circle to the first circle are perpendicular.

Hence, statement-1 is true and statement-2 explains it.

3. Statement -1 : Radical axis of the given circle is

$S_1 - S_2 = 0 \Rightarrow x + y - 7 = 0$

which passes through the centre of the second circle statement-1 is true.



Statement-2 is also true but it is not the explaination of statement-1.

4. Statement-1

$S_1 \equiv x^2 + y^2 - 4 = 0 \Rightarrow C_1\ (0, 0), \quad r_1 = 2$

$S_2 \equiv x^2+y^2-8x+7=0 \quad \Rightarrow C_2\ (4, 0), \quad r_2 = 3$

Now, $C_1C_2 = 4$

$r_1 + r_2 = 5$, $|r_1 - r_2| = 1$

$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\therefore$ circle intersect each other

Statement-2 is obviously false

Comprehension # 1

1. Let P be (h, k)

PA = nPB

$(h + 3)^2 + k^2 = n^2\ [(h - 3)^2 + k^2]$

$\therefore$ locus of P(h, k) is -

$x^2 + 6x + 9 + y^2 = n^2\ [x^2 - 6x + 9 + y^2]$

$x^2(1 - n^2) + y^2(1 - n^2) + 6x(1 + n^2) + 9(1 - n^2) = 0$

$$x^2 + y^2 + 6\ \frac{(1+n^2)}{1-n^2}x + 9 = 0\ \{\because\ n \neq 1\}$$

$\therefore$ Locus is a circle.

2. PA = PB when n = 1

$(h + 3)^2 + k^2 = (h - 3)^2 + k^2$

$h^2 + 6h + 9 + k^2 = h^2 - 6h + 9 + k^2$

$\therefore$ locus of P(h, k) is $x = 0$ $\therefore$ a straight line.

3. For $0 < n < 1$

locus is $(1 - n^2)(x^2 + y^2) + 6x(1 + n^2) + 9(1 - n^2) = 0$

putting A (–3, 0) in the above equation

$9(1 - n^2) - 18\ (1 + n^2) + 9(1 - n^2) = -\ 36n^2 < 0$

$\therefore$ A lies inside the circle.

Similarly for B (3, 0)

$9(1 - n^2) + 18\ (1 + n^2) + 9(1 - n^2)$

$= 36 > 0$

$\therefore$ B lies outside the circle.

4. for $n > 1$, locus is -

$(n^2 - 1)\ (x^2 + y^2) - 6x(1 + n^2) + 9(n^2 - 1) = 0$

putting A (– 3, 0) we get

$9(n^2 - 1) + 18(1 + n^2) + 9(n^2 - 1) = 36\ n^2 > 0$

& putting B(3, 0) we get

$9(n^2 - 1) - 18(1 + n^2) + 9(n^2 - 1) = -\ 36 < 0$

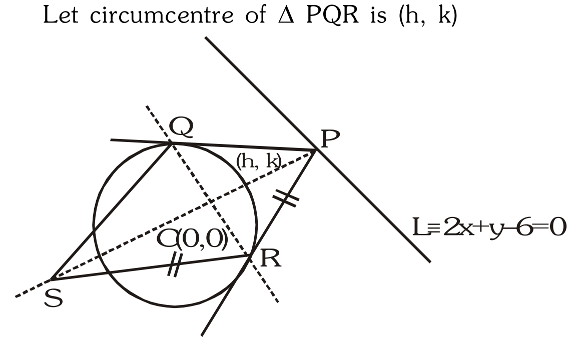
$\therefore$ A lies out side and B lies inside the circle.

5. We have seen whenever locus of P is a circle it never passes through A and B.

Comprehension # 2

1. Parallelogram PQSR is a rhombus

Let circumcentre of Δ PQR is (h, k)



which is the middle point of CP

$\therefore$ P becomes (2h, 2k) which satisfies the line $2x + y - 6 = 0$

$\therefore$ $2(2h) + 2k - 6 = 0$

$\therefore$ locus is $2x + y - 3 = 0$

2. If P(6, 8) then

Area (Δ PQR) = Area (Δ QRS)

$$\therefore \text{ Area } (\Delta \text{ PQR}) = \frac{RL^3}{R^2 + L^2}$$

$$= \frac{2.64.6\sqrt{6}}{100} = \frac{192\sqrt{6}}{25} \quad \{R = 2,\ L = 4\sqrt{6}\}$$

3. If P(3, 4) then

equation of chord of contact is

$3x + 4y - 4 = 0$ ... (1)

Straight line perpendicular to (1) & passing through centre of the circle is -

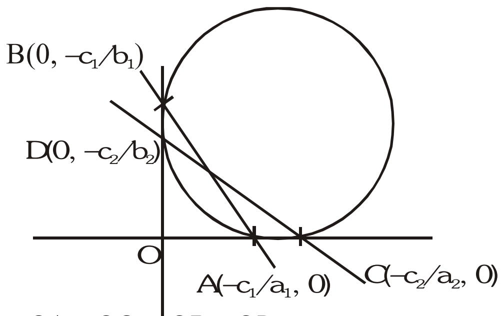
$4x - 3y = 0$ ... (2)

point of intersection of (1) & (2) is $\left(\frac{12}{25}, \frac{16}{25}\right)$

which is the middle point of PS

$\therefore$ coordinate of S are $\left(\frac{-51}{25}, \frac{-68}{25}\right)$

3. Since points A, B, C & D are concyclic

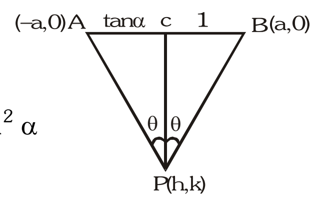


$\therefore$ OA . OC = OB . OD

$\Rightarrow \left(\frac{c_1}{a_1}\right)\left(\frac{c_2}{a_2}\right) = \left(\frac{c_2}{b_2}\right)\left(\frac{c_1}{b_1}\right)$

$\Rightarrow a_1a_2 = b_1b_2$

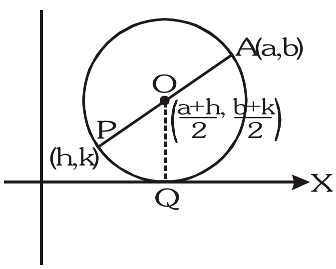
4. $\frac{AP}{PB} = \tan\alpha$



$\Rightarrow \frac{(h+a)^2+k^2}{(h-a)^2+k^2} = \tan^2\alpha$

simplifying we get the desired locus.

7.



AP = 2.OQ

$\sqrt{(h-a)^2+(k-b)^2} = 2 \cdot \frac{b+k}{2}$

$(h - a)^2 = (k + b)^2 - (k - b)^2$

$(h - a)^2 = 4bk$

$\therefore$ locus of P(h, k) is $(x - a)^2 = 4by$

10. Let the centre of the circle be (–r, r) where r is the radius of the circle

$\Rightarrow$ equation of circle will be :

$(x + r)^2 + (y - r)^2 = r^2$.

$\Rightarrow x^2 + 2rx + r^2 + y^2 - 2ry + r^2 = r^2$

$\Rightarrow x^2 + y^2 + 2rx - 2ry + r^2 = 0$

passes through (–2, 1)

$\Rightarrow r^2 - 6r + 5 = 0 \Rightarrow r = 1, 5$

when r = 1, $x^2 + 2x + y^2 - 2y + 1 = 0$

Hence A = 2, B = –2, C = 1

Also when r = 5

$x^2 + 10x + y^2 - 10y + 25 = 0$

$\Rightarrow$ A = 10, B = –10, C = 25

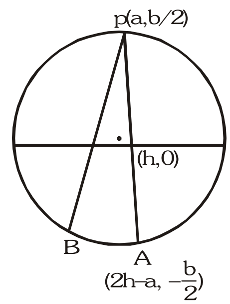
Hence the required triplets are (2,–2,1) & (10,–10,25)

$-\frac{A}{2} = -1, -5 \qquad \Rightarrow \qquad B = 2, -10$

Also $\sqrt{g^2+f^2-c} = r$

$\Rightarrow \frac{A^2}{4} + \frac{B^2}{4} - r^2 = C \Rightarrow C = 1, 25$

17.



C : $2x^2 + 2y^2 - 2ax - by = 0$

Point A (2h – a, – b/2) lies on the above circle.

$\therefore 2(2h - a)^2 + 2\frac{b^2}{4} - 2a(2h - a) - b(\frac{-b}{2}) = 0$

$2(4h^2 - 4ah + a^2) + \frac{b^2}{2} - 4ah + 2a^2 + \frac{b^2}{2} = 0$

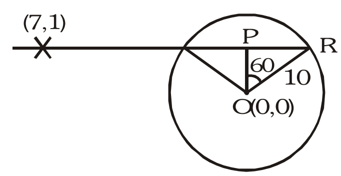
$8h^2 - 12ah + 4a^2 + b^2 = 0$

$\Rightarrow 144a^2 - 4.8\,(4a^2 + b^2) > 0 \quad [D > 0]$

$\Rightarrow 9a^2 - 8a^2 - 2b^2 > 0$

$\Rightarrow a^2 > 2b^2$

19.



Point of intersection of lines x – 2y – 5 = 0 & 7x + y = 50 will be (7, 1)

$\frac{OP}{OR} = \cos 60° = \frac{1}{2} \Rightarrow OP = 5$

Let the equation of PR be : (y – 1) = m(x – 7)

y – mx – 1 + 7m = 0

$OP = 5 = \left|\frac{-1+7m}{\sqrt{1+m^2}}\right|$

$25 + 25m^2 = 49m^2 + 1 - 14m$

$24m^2 - 14m - 24 = 0 \Rightarrow m = \frac{4}{3}, -\frac{3}{4}$

$\Rightarrow$ equation will be : $(y - 1) = \frac{4}{3}(x-7)$

& $(y - 1) = -\frac{3}{4}(x-7)$

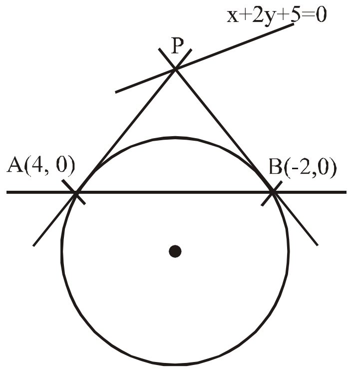
23. $x^2 + y^2 - 2x - 8 - 2\lambda y = 0 \Rightarrow S + \lambda L = 0$

$S : x^2 + y^2 - 2x - 8 = 0$

$L : y = 0$

Points of intersection of S = 0 & L = 0 are -

(4, 0) & (– 2, 0)



Let P be (h, k)

equation of chord of contact of P wrt given circle is

$hx + ky - 1\ (x + h) - \lambda(y + k) - 8 = 0$

$(h - 1)x + (k - \lambda)y - h - \lambda k - 8 = 0$

comparing with the line y = 0.

$$\frac{h-1}{0} = \frac{k-\lambda}{1} = \frac{-h-\lambda k-8}{0}$$

$h - 1 = 0 \quad \Rightarrow h = 1$

putting h = 1 in the line x + 2y + 5 = 0

$1 + 2k + 5 = 0 \quad \Rightarrow k = -3$

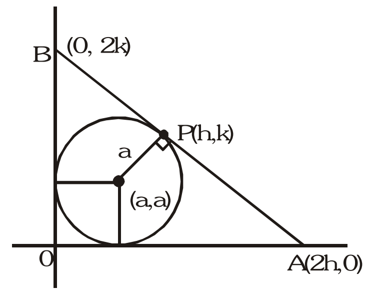
$-h - \lambda k - 8 = 0$

$-1 + 3\lambda - 8 = 0 \Rightarrow \lambda = 3$

∴ Equation of the required circle is -

$x^2 + y^2 - 2x - 6y - 8 = 0$

25. Δ AOB is right angled so its circumcentre is middle point of AB. Let it be P (h, k)



Equation of AB is $\frac{x}{2h} + \frac{y}{2k} = 1$

which is tangent to the give circle

$$\therefore \quad \left| \frac{\frac{a}{2h} + \frac{a}{2k} - 1}{\sqrt{\frac{1}{4h^2} + \frac{1}{4k^2}}} \right| = a$$

$(ak + ah - 2hk)^2 = a^2(h^2 + k^2)$

$a^2k^2 + a^2h^2 + 2a^2hk + 4h^2k^2 - 4hk(ah+ak) = a^2(h^2+k^2)$

∴ locus of P(h, k) is $a^2 + 2xy - 2(ax + ay) = 0$

27. The given circles are

$S_1 = x^2 + y^2 + 4x - 6y + 9 = 0$

$S_2 = x^2 + y^2 - 5x + 4y + 2 = 0$

& variable circle is

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

Now, S & S_1 are orthogonal

∴ $\quad 4g - 3f = c + 9$ .....(1)

S & S_2 are also orthogonal

∴ $\quad -5g + 4f = c + 2$ ......(2)

(1) – (2)

$9g - 10f = 7$

∴ locus of (– g, – f) is

$-9x + 10y = 7$

$9x - 10y = -7$

$9x - 10y + 7 = 0$

which is the radial axis of the two given circles.

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

2. Let P be (x_1, y_1)

Coordinates of any point on the curve at a distance r from P are $(x_1 + r\cos\theta,\ y_1 + r\sin\theta)$

$a(x_1 + r\cos\theta)^2 + 2h(x_1 + r\cos\theta)(y_1 + r\sin\theta) + b(y_1 + r\sin\theta)^2 = 1$

$\Rightarrow r^2(a\cos^2\theta + 2h\sin\theta\cos\theta + b\sin^2\theta) + 2r(ax_1\cos\theta + hx_1\sin\theta + hy_1\cos\theta + by_1\sin\theta) + ax_1^2 + 2hx_1y_1 + by_1^2 - 1 = 0$

which is quadratic in 'r'

$$\therefore\ r_1 r_2 = \frac{ax_1^2 + 2hx_1y_1 + by_1^2 - 1}{a\cos^2\theta + h\sin 2\theta + b\sin^2\theta}$$

$$PQ\ .\ PR = \frac{ax_1^2 + 2hx_1y_1 + by_1^2 - 1}{a + (b-a)\sin^2\theta + h\sin 2\theta}$$

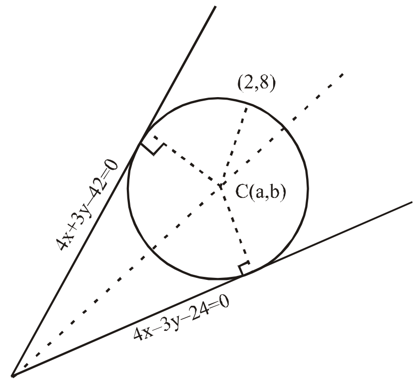
PQ . PR will be independent of θ if

$b - a = 0 \quad \& \quad h = 0$

$\Rightarrow a = b \quad \& \quad h = 0$

Hence, in this condition curve becomes a circle.

9. Let the centre of the circle be (a, b)



$(a-2)^2 + (b-8)^2 = r^2$ ... (1)

$\left|\dfrac{4a-3b-24}{5}\right| = \left|\dfrac{4a+3b-42}{5}\right| = r$ ... (2)

$\Rightarrow 4a - 3b - 24 = \pm(4a + 3b - 42)$ ... (3)

$6b = 18 \Rightarrow b = 3$

from (1) $(a-2)^2 = r^2 - 25$

from (2) $4a - 3b - 24 = 5r$

$4a = 5r + 33$

$(a-2)^2 = \left(\dfrac{4a-33}{5}\right)^2 - 25$

$(a-2)^2 = \left(\dfrac{4a-33}{5}-5\right)\left(\dfrac{4a-33}{5}+5\right)$

$(a-2)^2 = \left(\dfrac{4a-58}{5}\right)\left(\dfrac{4a-8}{5}\right)$

$25(a-2)^2 = 4.2\,(2a-29)(a-2)$

$25(a-2) = 8(2a-29)$ or $a - 2 = 0$

$\Rightarrow a = 2$

Also $9a = -182$

$a = \dfrac{-182}{9}$

$\therefore a = 2, b = 3, r = 5$

& $a = \dfrac{-182}{9}$, $b = 3$, $r = \dfrac{205}{9}$

(from –ve sign in (3)) $8a = 66$

$\Rightarrow a = 33/4 > 8$ (rejected)

11. The parametric form of OP is $\dfrac{x-0}{\cos 45°} = \dfrac{y-0}{\sin 45°}$

Since, $OP = 4\sqrt{2}$

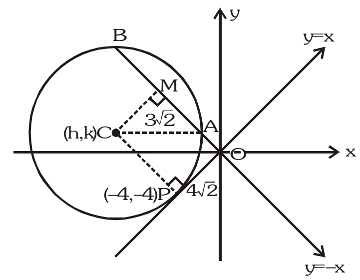
So, the coordinates of P are given by

$\dfrac{x-0}{\cos 45°} = \dfrac{y-0}{\sin 45°} = -4\sqrt{2}$

So, P(–4, –4)

Let, C(h, k) be the centre of circle and r be its radius,

Now, CP ⊥ OP



$\Rightarrow \dfrac{k+4}{h+4}.(1) = -1$

$\Rightarrow h + k = -8$ .....(i)

Also, $CP^2 = (h+4)^2 + (k+4)^2$

$\Rightarrow (h+4)^2 + (k+4)^2 = r^2$ .....(ii)

In ΔACM, we have $AC^2 = (3\sqrt{2})^2 + \left(\dfrac{h+k}{\sqrt{2}}\right)^2$

$\Rightarrow r^2 = 18 + 32$

$\Rightarrow r = 5\sqrt{2}$ ......(iii)

also, CP = r

$\Rightarrow \left|\dfrac{h-k}{\sqrt{2}}\right| = r$

$\Rightarrow h - k = \pm 10$ ....(iv)

From (i) and (iv), we get

(h = –9, k = 1) or (h = 1, k = –9)

Thus, the equation of the circles are

$(x+9)^2 + (y-1)^2 = (5\sqrt{2})^2$

and $(x-1)^2 + (y+9)^2 = (5\sqrt{2})^2$

or $x^2 + y^2 + 18x - 2y + 32 = 0$

and $x^2 + y^2 - 2x + 18y + 32 = 0$

Clearly, (–10, 2) lies interior of

$x^2 + y^2 + 18x - 2y + 32 = 0$

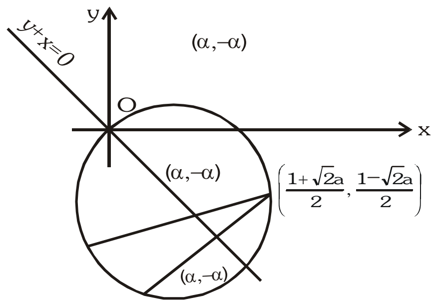
Hence, the required equation of circle is

$x^2 + y^2 + 18x - 2y + 32 = 0$

12. $2x^2 + 2y^2 - (1+\sqrt{2}a)x - (1-\sqrt{2}a)y = 0$

$\Rightarrow x^2 + y^2 - \left(\dfrac{1+\sqrt{2}a}{2}\right)x - \left(\dfrac{1-\sqrt{2}a}{2}\right)y = 0$

Since, y + x = 0 bisects two chords of this circle, mid-points of the chords must be of the form (α, –α)



Equation of the chord having (α, −α) as mid-points is T = S_1

$$\Rightarrow\ x\alpha+y(-\alpha)-\left(\frac{1+\sqrt{2}a}{4}\right)(x+\alpha)-\left(\frac{1-\sqrt{2}a}{4}\right)(y-\alpha)$$

$$=\alpha^2+(-\alpha)^2-\left(\frac{1+\sqrt{2}a}{2}\right)\alpha-\left(\frac{1-\sqrt{2}a}{2}\right)(-\alpha)$$

$$\Rightarrow\quad 4x\alpha-4y\alpha-(1+\sqrt{2}a)x-(1+\sqrt{2}a)\alpha$$
$$-(1-\sqrt{2}a)y+(1-\sqrt{2}a)\alpha$$
$$=4\alpha^2+4\alpha^2-(1+\sqrt{2}a).2\alpha+(1-\sqrt{2}a).2\alpha$$

$$\Rightarrow\quad 4\alpha x-4\alpha y-(1+\sqrt{2}a)x-(1-\sqrt{2}a)y$$
$$=8\alpha^2-(1+\sqrt{2}a)\alpha+(1-\sqrt{2}a)\alpha$$

But this chord will pass through the point

$$\left(\frac{1+\sqrt{2}a}{2},\frac{1-\sqrt{2}a}{2}\right)$$

$$\therefore\quad 4\alpha\left(\frac{1+\sqrt{2}a}{2}\right)-4\alpha\left(\frac{1-\sqrt{2}a}{2}\right)$$
$$-\frac{(1+\sqrt{2}a)(1+\sqrt{2}a)}{2}-\frac{(1-\sqrt{2}a)(1-\sqrt{2}a)}{2}$$
$$=8\alpha^2-2\sqrt{2}a\alpha$$

$$\Rightarrow\quad 2\alpha[(1+\sqrt{2}a-1+\sqrt{2}a)]=8\alpha^2-2\sqrt{2}a\alpha$$

$$\Rightarrow\quad 4\sqrt{2}a\alpha-\frac{1}{2}[2+2(\sqrt{2}a)^2]=8\alpha^2-2\sqrt{2}a\alpha$$

[∵ $(a+b)^2+(a-b)^2=2a^2+2b^2$]

$$\Rightarrow\quad 8\alpha^2-6\sqrt{2}a\alpha+1+2a^2=0$$

But this quadratic equation will have two distinct roots if $(6\sqrt{2}a)^2-4(8)(1+2a^2)>0$

$\Rightarrow\quad 72a^2-32(1+2a^2)>0$

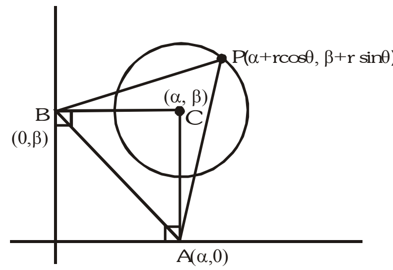
$\Rightarrow\quad 72a^2-32-64a^2>0\Rightarrow 8a^2-32>0$

$\Rightarrow\quad a^2>4\quad\Rightarrow\quad a<-2\cup a>2$

Therefore, $a\in(-\infty,-2)\cup(2,\infty)$.

14. Let the equation of the circle be

$(x-\alpha)^2+(y-\beta)^2=r^2$



coordinates of P are

∴ $(\alpha+r\cos\theta,\ \beta+r\sin\theta)$

Let centroid of Δ PAB be (h, k)

$3h=\alpha+\alpha+r\cos\theta\Rightarrow r\cos\theta=3h-2\alpha$

$3k=\beta+\beta+r\sin\theta\Rightarrow r\sin\theta=3k-2\beta$

squaring and adding

$(3h-2\alpha)^2+(3k-2\beta)^2=r^2$

∴ locus of (h, k) is

$$\left(x-\frac{2\alpha}{3}\right)^2+\left(y-\frac{2\beta}{3}\right)^2=\frac{r^2}{9}$$

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. Length of tangent

$=\sqrt{3^2+(-4)^2-4(3)-6(-4)+3}=\sqrt{40}$

∴ Square of length of tangent = 40

3. When two circles intersect each other, then

Difference between their radii < Distance between centers $\Rightarrow r-3<5\Rightarrow r<8$ ... (i)

Sum of their radii > Distance between centres ...(ii)

$\Rightarrow r+3>5\Rightarrow r>2$

Hence by (i) and (ii) $2<r<8$

4. Centre of circle = Point of intersection of diameters = (1, −1)

Now area = 154 $\Rightarrow \pi r^2=154\Rightarrow r=7$

Hence the equation of required circle is

$(x-1)^2+(y+1)^2=7^2\Rightarrow x^2+y^2-2x+2y=47$

5. Let the variable circle be

$x^2+y^2+2gx+2fy+c=0$ ..... (i)

Circle (i) cuts circle $x^2+y^2-4=0$ orthogonally

$\Rightarrow 2g.0+2f.0=c-4\Rightarrow c=4$

Since circle (i) passes through (a, b)

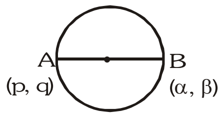
∴ $a^2+b^2+2ga+2fb+4=0$

$\therefore$ Locus of centre (–g, –f) is

$2ax + 2by - (a^2 + b^2 + 4) = 0$

6. Equation of circle having AB as diameter is

$(x - p)(x - \alpha) + (y - q)(y - \beta) = 0$



or $x^2 + y^2 - (p + \alpha)x - (q + \beta)y + p\alpha + q\beta = 0$ ..... (i)

as it touches x-axis putting y = 0,

we get $x^2 - (p + \alpha)x + p\alpha + q\beta = 0$ ..... (ii)

Since, circle (i) touches x-axis

Discriminant of equation (ii) = 0

$\Rightarrow (p + \alpha)^2 = 4(p\alpha + q\beta) \Rightarrow (p - \alpha)^2 = 4q\beta$

$\therefore$ Locus of B(α, β) is $(p - x)^2 = 4qy$

or $(x - p)^2 = 4qy$

7. According to question two diameters of the circle are

$2x + 3y + 1 = 0$ and $3x - y + 4 = 0$

Solving, we get x = 1, y = –1

$\therefore$ Centre of the circle is (1, –1)

Given $2\pi r = 10\pi \Rightarrow r = 5$

$\therefore$ Required circle is $(x - 1)^2 + (y + 1)^2 = 5^2$

or $x^2 + y^2 - 2x + 2y - 23 = 0$

8. Given, circle is $x^2 + y^2 - 2x = 0$ ..... (i)

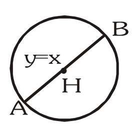
and line is y = x ..... (ii)

Puting y = x in (i),

We get $2x^2 - 2x = 0 \Rightarrow x = 0, 1$

From (i), y = 0, 1

Let A = (0, 0), B = (1, 1)



Equation of required circle is

$(x - 0)(x - 1) + (y - 0)(y - 1) = 0$

or $x^2 + y^2 - x - y = 0$

9. Equation of line PQ (i.e. common chord) is

$5ax + (c - d)y + a + 1 = 0$ ..... (i)

Also given equation of line PQ is

$5x + by - a = 0$ ..... (ii)

Therefore $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$; As $\frac{a+1}{-a} = a$

$\Rightarrow a^2 + a + 1 = 0$

Therefore no real value of a exists, (as D < 0)

10. Let centre ≡ (h, k); As $C_1C_2 = r_1 + r_2$, (Given)

$\Rightarrow \sqrt{(h-0)^2 + (k-3)^2} = |k + 2|$

$\Rightarrow h^2 = 5(2k - 1)$

Hence locus, $x^2 = 5(2y - 1)$, which is parabola

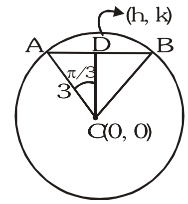
14. Let AB be the chord subtending angle 2π/3 at the centre C of circle

Now, $\angle ACD = \pi/3$

Let the coordinates of midpoint D be (h, k)

In ΔACD, $\cos\frac{\pi}{3} = \frac{CD}{CA}$

$\Rightarrow \frac{1}{2} = \frac{\sqrt{h^2+k^2}}{3}$



$\Rightarrow x^2 + y^2 = \frac{9}{4}$, which is the required locus.

15. Equation of circle $(x - h)^2 + (y - k)^2 = k^2$

It is passing through (–1, 1) then

$(-1 - h)^2 + (1 - k)^2 = k^2$ $h^2 + 2h - 2k + 2 = 0$

$D \geq 0$ $2k - 1 \geq 0 \Rightarrow k \geq 1/2$

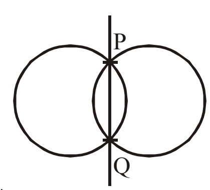
17. Let A, B, C are represented by the point (x, y)

$$\frac{\sqrt{(x-1)^2 + y^2}}{\sqrt{(x+1)^2 + y^2}} = \frac{1}{2}$$

$8x^2 + 8y^2 - 20x + 8 = 0$

Which is the circle which passes through the points A, B, C then circumcentre will be the centre of the circle $\left(\frac{5}{4}, 0\right)$.

18. $Eq^{n.}$ of line PQ



$x + 5y + 2p - 5 + p^2 = 0$

P, Q and (1, 1)

will not lie on a circle of (1, 1)

Lies on the line

$x + 5y + p^2 + 2p - 5 = 0$

$\Rightarrow 1 + 5 + p^2 + 2p - 5 = 0$

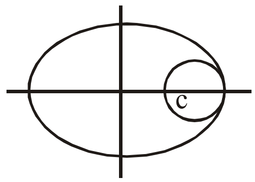
$p^2 + 2p + 1 = 0$

$\Rightarrow p = -1$

Therefore their is a circle passing through P, Q and (1, 1) for all values of p.

Except p = – 1.

21.



$\left|\frac{a}{2}\right| = c - \left|\frac{a}{2}\right|$

$|a| = C$

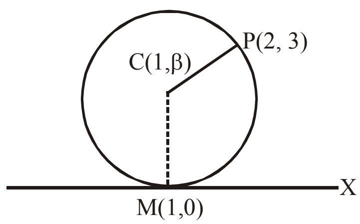
22. (1, 0) and (0, 1) will be ends of diameter

So equation of circle

$(x - 1)(x - 0) + (y - 0)(y - 1)$

$x^2 + y^2 - x - y = 0$

23.



Let center of the circle be $C(1, \beta)$

$\beta^2 = (2 - 1)^2 + (3 - \beta)^2$

$\Rightarrow \beta^2 = -6\beta + 10 + \beta^2$

$\Rightarrow \beta = \frac{5}{3}$

$\therefore r = \frac{5}{3}$

diameter $= \frac{10}{3}$

24. Let equation of circle be $(x - 3)^2 + (y + r)^2 = r^2$ $\because$ it passes through $(1, -2)$

$\Rightarrow$ $r = 2$

$\Rightarrow$ circle is $(x - 3)^2 + (y + 2)^2 = 4$

$\Rightarrow$ $(5, -2)$

Aliter :

$(x - 3)^2 + y^2 + \lambda y = 0$ ....(1)

Putting $(1, -2)$ in (1)

$\Rightarrow$ $\lambda = 4$

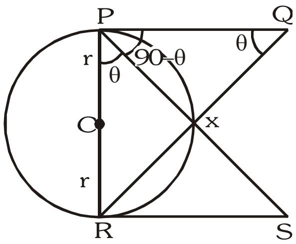
Required circle is

$x^2 + y^2 - 6x + 4y + 9 = 0$

point $(5, -2)$ satisfies the equation the equation

1. Let $\angle RPS = \theta$

$\angle XPQ = 90 - \theta$



$\therefore$ $\angle PQX = \theta$ ($\because$ $\angle PXQ = 90$)

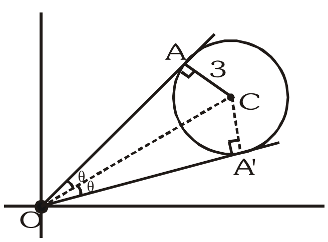
$\therefore$ $\Delta PRS \sim \Delta QPR$ (AAA similarity)

$\therefore$ $\frac{PR}{QP} = \frac{RS}{PR}$ $\Rightarrow$ $PR^2 = PQ.RS$

$\Rightarrow$ $PR = \sqrt{PQ.RS}$

2. The equation $2x^2 - 3xy + y^2 = 0$ represents pair of tangents OA and OA'.

Let angle between these to tangents be 2θ.



Then $\tan 2\theta = \frac{2\sqrt{\left(\frac{-3}{2}\right)^2 - 2\times 1}}{2+1}$

[Using $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$]

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{3}$ $\Rightarrow \tan^2\theta + 6\tan\theta - 1 = 0$

$\tan\theta = \frac{-6 \pm \sqrt{36+4}}{2} = -3 \pm \sqrt{10}$

As θ is acute $\therefore$ $\tan\theta = \sqrt{10} - 3$

Now we know that line joining the point through which tangents are drawn to the centre bisects the angle between the tangents,

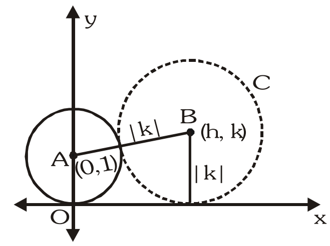
$\therefore$ $\angle AOC = \angle A'OC = \theta$

In ΔOAC $\tan\theta = \frac{3}{OA}$

$\Rightarrow OA = \frac{3}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$

$\therefore$ $OA = 3(3 + \sqrt{10})$

9. Let the centre of circle C be (h, k). Then as this circle touches axis of x its radius = $|k|$



Also it touches the given circle $x^2 + (y - 1)^2 = 1$, centre (0, 1) radius 1, externally

Therefore

The distance between centres = sum of radii

$\Rightarrow$ $\sqrt{(h-0)^2 + (k-1)^2} = 1 + |k|$

$\Rightarrow$ $h^2 + k^2 - 2k + 1 = (1 + |k|)^2$

$\Rightarrow$ $h^2 + k^2 - 2k + 1 = 1 + 2|k| + k^2$

$\Rightarrow$ $h^2 = 2k + 2|k|$

$\therefore$ Locus of (h, k) is, $x^2 = 2y + 2|y|$

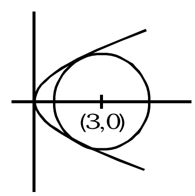
Now if $y > 0$, it becomes $x^2 = 4y$

and if $y \le 0$, it becomes $x = 0$

$\therefore$ Combining the two, the required locus is

$\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

12 $C_1 : y^2 = 4x$ $C_2 : x^2 + y^2 - 6x + 1 = 0$



$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$ $\Rightarrow x = 1$

$y = \pm 2$

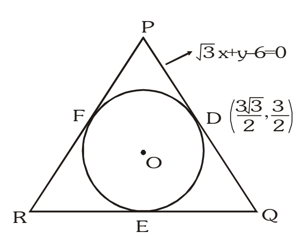
so the curves touches each other at two points (1, 2) & (1, –2)

13. Eq. of circle is $(x + 3)^2 + (y - 5)^2 = 4$

Distance between the given lines = $\frac{6}{\sqrt{13}}$ < radius

So S(II) is false & S(I) is true

14. (i) $m_{PQ} = -\sqrt{3}$



so slope of OD = $\frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \quad \dfrac{x-\frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{y-\frac{3}{2}}{\frac{1}{2}} = \pm 1$

$(2\sqrt{3},2)$(not possible) & $(\sqrt{3},1)$

hence circle is $(x-\sqrt{3})^2+(y-1)^2=1$

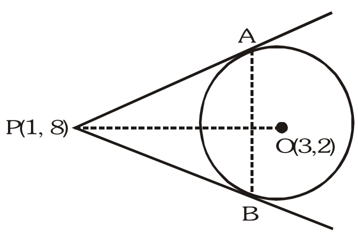
(ii) For point E $\dfrac{x-\sqrt{3}}{-\frac{\sqrt{3}}{2}} = \dfrac{y-1}{\frac{1}{2}} = 1 \quad \left[\therefore\ E\left(\dfrac{\sqrt{3}}{2},\dfrac{3}{2}\right)\right]$

For point F $\dfrac{x-\sqrt{3}}{0} = \dfrac{y-1}{-1} = 1 \quad \left[\therefore\ F\left(\sqrt{3},0\right)\right]$

(iii) Equation of line RP $\quad y = 0$

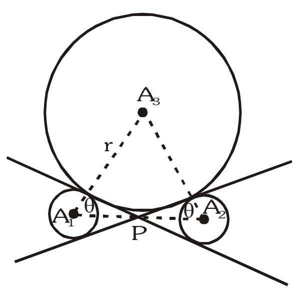
Equation of line QR $\quad y - \dfrac{3}{2} = \sqrt{3}\left(x-\dfrac{\sqrt{3}}{2}\right)$

$$y = \sqrt{3}\,x$$

15. 

The required circle is a circle described on OP as diameter.

16. **Ans. 8**



In triangle $A_1A_2A_3$

$A_1A_3 = A_3A_2$

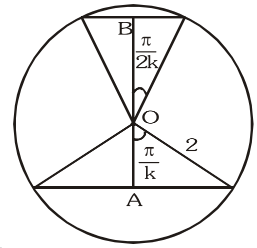
Let angle $A_3A_1A_2 = \theta$, $\cos\theta = \dfrac{1}{3}$, $\sin\theta = \dfrac{2\sqrt{2}}{3}$

Apply sine rule in triangle $A_1A_2A_3$

$\dfrac{6}{\sin(\pi-2\theta)} = \dfrac{r+1}{\sin\theta}$

$\Rightarrow r = 8$

17. $OA = 2\cos\dfrac{\pi}{k}$



$OB = 2\cos\dfrac{\pi}{2k}$

$2\cos\dfrac{\pi}{k} + 2\cos\dfrac{\pi}{2k} = \sqrt{3}+1$

$2\cos^2\dfrac{\pi}{2k} - 1 + \cos\dfrac{\pi}{2k} = \dfrac{\sqrt{3}+1}{2}$

Let $\cos\dfrac{\pi}{2k} = t$

$2t^2 + t - 1 - \dfrac{\sqrt{3}+1}{2} = 0$

$\Rightarrow \quad 4t^2 + 2t - (3+\sqrt{3}) = 0$

$\Rightarrow \quad t = \dfrac{\sqrt{3}}{2}, -\dfrac{1+\sqrt{3}}{2}$

$t = -\dfrac{1+\sqrt{3}}{2}$ (not possible)

$t = \dfrac{\sqrt{3}}{2} = \cos 30° = \cos\dfrac{\pi}{6} \quad \Rightarrow \quad \cos\dfrac{\pi}{2k} = \cos\dfrac{\pi}{6}$

$k = 3$

18. Family of circle which touches y-axis at (0,2) is

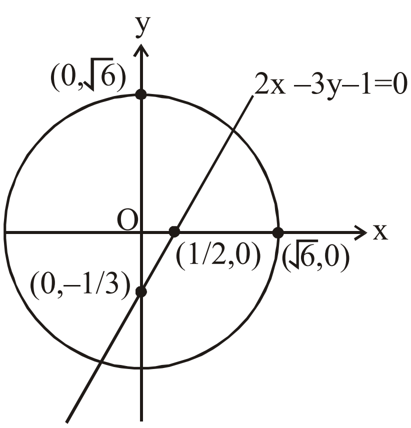
$x^2 + (y-2)^2 + \lambda x = 0$

Passing through (–1,0)

$\Rightarrow \quad 1 + 4 - \lambda = 0 \quad \Rightarrow \quad \lambda = 5$

$\therefore \quad x^2 + y^2 + 5x - 4y + 4 = 0$

which satisfy the point (–4,0).

19. 

If the point lies inside the smaller part, then origin and point should give opposite signs w.r.t. line & point

should lie inside the circle.

for origin : 2 0 – 3 0 – 1 = – 1 (–ve)

for $(2, \dfrac{3}{4})$:2 2 – 3 $\dfrac{3}{4}$ –1

$= \dfrac{3}{4}$ (+ve); point lies inside the circle for $\left(\frac{5}{2},\frac{3}{4}\right):2\ \frac{5}{2}-3\ \frac{3}{4}-1=\frac{7}{4}$ (+ve) ; point lies outside the circle

For $\left(\frac{1}{4},-\frac{1}{4}\right):2\ \frac{1}{4}-3\left(-\frac{1}{4}\right)-1=\frac{1}{4}$ (+ve) ; point lies inside the circle

For $\left(\frac{1}{8},\frac{1}{4}\right):2\ \frac{1}{8}-3\left(\frac{1}{4}\right)-1=\frac{-3}{2}$ (–ve) ; point lies inside the circle.

$\therefore$ 2 points lie inside smaller part.

20. Let mid point be (h, k),

then chord of contact :

$hx + ky = h^2 + k^2$ ....(i)

Let any point on the line 4x – 5y = 20 be $\left(x_1,\frac{4x_1-20}{5}\right)$

$\therefore$ Chord of contact :

$5x_1x + (4x_1 - 20)y = 45$..........(ii)

(i) and (ii) are same

$\therefore\ \frac{5x_1}{h}=\frac{4x_1-20}{k}=\frac{45}{h^2+k^2}$

$\Rightarrow\ x_1=\frac{9h}{h^2+k^2}$

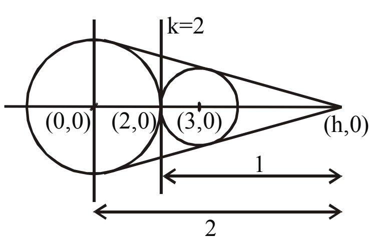
and $x_1=\frac{45k+20(h^2+k^2)}{4(h^2+k^2)}$

$\Rightarrow\ \frac{9h}{h^2+k^2}=\frac{45k+20(h^2+k^2)}{4(h^2+k^2)}$

$\Rightarrow\ 20(h^2 + k^2) - 36h + 45k = 0$

$\therefore$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

21. $h=\frac{2\times3-1\times0}{2-1}=6$



equation of tangents from (6, 0) :

y – 0 = m(x – 6) $\Rightarrow$ y – mx + 6m = 0

use p = r

$\left|\frac{6m}{\sqrt{1+m^2}}\right|=2\ \Rightarrow\ 36m^2 = 4 + 4m^2$

$32m^2 = 4$

$m^2 = 1/8$

$\Rightarrow\ m=\pm\frac{1}{2\sqrt{2}}$

at $m=-\frac{1}{2\sqrt{2}}$

equation of tangent will be $x+2\sqrt{2}y=6$

22. Equation of tangent at P will be $\sqrt{3}x+y=4$

Slope of line L will be $\frac{1}{\sqrt{3}}$

Let equation of L be : $y=\frac{x}{\sqrt{3}}+c$

$\Rightarrow\ x-\sqrt{3}y+\sqrt{3}c=0$

Now this L is tangent to 2nd circle

So $\frac{3+\sqrt{3}c}{2}=\pm1\ \Rightarrow\ c=-\frac{1}{\sqrt{3}}$

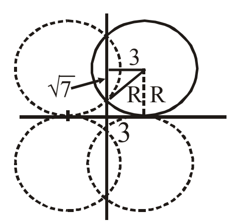
or $c=-\frac{5}{\sqrt{3}}$

using $c=-\frac{1}{\sqrt{3}}$

$y=\frac{x}{\sqrt{3}}-\frac{1}{\sqrt{3}}\ \Rightarrow\ x-\sqrt{3}y=1$. Hence (A)

23. As per figure,

$R^2 = 3^2+\left(\sqrt{7}\right)^2$



$\Rightarrow$ R = 4

$\therefore$ centre ≡ (3,4)

radius 4

$\therefore$ equation $x^2 + y^2 - 6x - 8y + 9 = 0$

such a circle can lie in all 4 quadrants as shown in figure.

$\therefore$ equation can be $x^2 + y^2 \pm 6x \pm 8y + 9 = 0$