EXERCISE - 01 CHECK YOUR GRASP

3. Sum of interior angles of a n sided polygon
$=(n-2)\quad 180$

$= \frac{n}{2}[240+(n-1)5] \qquad \Rightarrow \qquad n = 9,\ 16$

n = 16 is to be rejected.

$(T_{16}=120 + 15 \quad 5 = 195 > 180)$

5. Horizontal $1+\frac{1}{4}+\frac{1}{16}+.....\infty = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

Vertical $\frac{1}{2}+\frac{-1}{8}+\frac{1}{32}.....\infty = \frac{\frac{1}{2}}{1+\frac{1}{4}} = \frac{2}{5}$

10. $\Rightarrow \quad \frac{a+rd}{a+nd} = \frac{a+nd}{a+md}$

$\Rightarrow \quad \frac{1+r(d/a)}{1+n(d/a)} = \frac{1+n(d/a)}{1+m(d/a)}$ Let $\frac{d}{a}=x$

$\Rightarrow \quad (1+nx)^2 = (1+rx)(1+mx)$

$\Rightarrow \quad (n^2-mr)x^2 + (2n-r-m)x = 0 \Rightarrow x=0$

or $x = -\left(\frac{2n-r-m}{n^2-mr}\right) = \left(\frac{2n-r-m}{\frac{n(m+r)}{2}-n^2}\right) = \frac{-2}{n}$

(m, n, r are in H.P. $\Rightarrow$ $mr = \frac{n(m+r)}{2}$)

14. $a, A_1, ...\ A_n, b.$ $\qquad d = \frac{b-a}{n+1}$

$p = A_1 = a + \frac{b-a}{n+1} = \frac{an+a+b-a}{n+1} = \frac{an+b}{n+1}$

15. Given $\frac{1}{a}+\frac{1}{a-2b}+\frac{1}{c}+\frac{1}{c-2b} = 0$

$\Rightarrow \quad \left(\frac{1}{a}+\frac{1}{c-2b}\right)+\left(\frac{1}{c}+\frac{1}{a-2b}\right) = 0$

$\Rightarrow \quad (a+c-2b)\left(\frac{1}{a(c-2b)}+\frac{1}{c(a-2b)}\right) = 0$

As $\quad a+c-2b \neq 0$

$\Rightarrow \quad \frac{1}{b} = \frac{1}{a}+\frac{1}{c}$

i.e. $\quad$ a, 2b, c are in H.P.

18. $\sum_{s=1}^{n}\left\{\sum_{r=1}^{s} r\right\} = \sum_{s=1}^{n}\frac{s(s+1)}{2}$

$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]$

$= \frac{n(n+1)}{4}\left[\frac{2n+1+3}{3}\right] = \frac{1}{12}n(n+1)(2n+4)$

$\Rightarrow \quad a = \frac{1}{6},\ b = \frac{1}{2},\ c = \frac{1}{3}$

19. We have $b^2 = ac$ .... (i)

and $\quad 2\log\left(\frac{3b}{5c}\right) = \log\left(\frac{5c}{a}\right)+\log\left(\frac{a}{3b}\right)$

$= \log\left(\frac{5c}{a}.\frac{a}{3b}\right) = \log\left(\frac{5c}{3b}\right) = -\log\left(\frac{3b}{5c}\right)$

$\Rightarrow \quad 3\log\left(\frac{3b}{5c}\right) = 0 \ \Rightarrow\ b = \frac{5}{3}c$ .... (ii)

From (i) & (ii), we have $a = \frac{b^2}{c} = \frac{25c}{9}$

Now, we have $b+c = \frac{5c}{3}+c = \frac{8c}{3} < \frac{25c}{9} = a$

Hence a, b, c cannot form the sides of a triangle.

23. $x = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$

$y = \frac{1}{1-\sin^2\theta} = \frac{1}{\cos^2\theta}$

$z = \frac{1}{1-\sin^2\theta\cos^2\theta}$

above equation satisfy option (B) & (C).

EXERCISE - 02 BRAIN TEASERS

1. $\frac{S_{kx}}{S_x} = \frac{\frac{kx}{2}[2a+(kx-1)d]}{\frac{x}{2}[2a+(x-1)d]} = \frac{k[2a+(kx-1)d]}{[2a+(x-1)d]}$

$= \frac{k[(2a-d)+kxd]}{[(2a-d)+xd]}$

Now $\frac{S_{kx}}{S_x}$ is independent of x if $2a - d = 0$

$\Rightarrow a = \frac{d}{2}$

2. $\alpha + \beta + \gamma = -3a, \qquad \alpha\,\beta\,\gamma = -c$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3b \Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{3b}{\alpha\beta\gamma}$$

$$\Rightarrow \frac{2}{\beta}+\frac{1}{\beta} = \frac{3b}{-c} \qquad (\because \alpha, \beta, \gamma \text{ in H.P})$$

$$\Rightarrow \frac{1}{\beta} = -\frac{b}{c} \qquad \Rightarrow \beta = \frac{-c}{b}$$

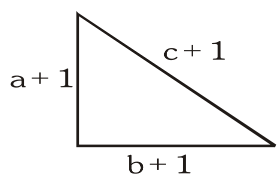
5. $a_1 + a_{10} = a_2 + a_9 = ... = (a + b)$

$g_1 g_{10} = g_2\, g_9 = ... = ab$

$$\Rightarrow \frac{5(a+b)+4(a+b)+3(a+b)+2(a+b)+(a+b)}{ab}$$

$$= 15\frac{(a+b)}{ab} = \frac{30}{h} \qquad \left(\because h = \frac{2ab}{a+b}\right)$$

7. $\sum_{r=1}^{n}(2r-1) = n^2$



$\Rightarrow a = 2n - 1$

$$\Rightarrow \quad n = \frac{a+1}{2}$$

$\Rightarrow \quad (a + 1)^2 + (b + 1)^2 = (c + 1)^2$

$\quad = 8 \qquad = 6 \qquad = 10 \quad (\because \; a+b+c = 21)$

$\Rightarrow \quad a = 7 \qquad b = 5 \qquad c = 9$

Hence G = 9 $\qquad$ L = 5

$\qquad$ G – L = 4 & $\quad$ a – b = 2

11. G.M. $\ge$ H.M.

$$(a.b.b.c.c.c)^{\frac{1}{6}} \ge \frac{6}{\frac{1}{a}+\frac{1}{b}+\frac{1}{b}+\frac{1}{c}+\frac{1}{c}+\frac{1}{c}}$$

$$\Rightarrow (64)^{1/6} \ge \frac{6}{\frac{1}{a}+\frac{2}{b}+\frac{3}{c}} \qquad \Rightarrow \frac{1}{a}+\frac{2}{b}+\frac{3}{c} \ge 3$$

12. Using AM $\ge$ GM,

$$\frac{1+a_1+a_1^2}{3} \ge (1.a_1.a_1^2)^{1/3} \Rightarrow 1+a_1+a_1^2 \ge 3a_1$$

$$\Rightarrow 1+a_2+a_2^2 \ge 3a_2 \Rightarrow ..... \Rightarrow 1+a_n+a_n^2 \ge 3a_n$$

Multiplying these,

$$(1+a+a_1^2)....(1+a_n+a_n^2) \ge 3^n(a_1a_2a_3....a_n) = 3^n.1$$

13. Let a_1, $a_1 + d_1$, $a_1 + 2d_1$,........ and b_1, $b_1 + d_2$, $b_1 + 2d_2$,........ be two A.P.'s

$\therefore \quad a_{100} = a_1 + 99d_1,\ b_{100} = b_1 + 99d_2$

Adding $\quad a_{100} + b_{100} = a_1 + b_1 + 99(d_1 + d_2)$

or $\quad 100 = 100 + 99(d_1 + d_2)$

$\Rightarrow \quad d_1 + d_2 = 0$ or $d_1 = -d_2$

$\therefore \quad$ option (B) gives $a_n + b_n$

$= a_1 + (n - 1)d_1 + b_1 + (n - 1)d_2$

$= a_1 + b_1 = 100$

option (C) is obviously true.

$$\text{Now} \quad \sum_{r=1}^{100}(a_r + b_r) = 100(a_1+b_1) = 10^4$$

18. $a + b + c = xb$

$$\text{Divide by b,} \quad \frac{a}{b}+1+\frac{c}{b} = x$$

or $\quad \frac{1}{r}+1+r = x \quad$ where r is common ratio of G.P.

$\Rightarrow \quad r^2 + r(1 - x) + 1 = 0$

since r is real & distinct $\quad \Rightarrow \quad D > 0$

$\therefore \quad (1 - x)^2 - 4 > 0 \quad \Rightarrow \quad x^2 - 2x - 3 > 0$

or $\quad (x + 1)(x - 3) > 0 \quad \Rightarrow \quad x > 3$ or $x < -1$

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Fill in the blanks :

4. $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}........\infty = \frac{\pi^2}{6}$

$$\text{Now} \quad \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}........$$

$$= \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}...... - \left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}........\infty\right)$$

$$= \frac{\pi^2}{6} - \frac{1}{2^2}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}........\infty\right)$$

$$= \frac{\pi^2}{6} - \frac{1}{4}.\frac{\pi^2}{6} = \frac{\pi^2}{8}$$

6. $T_n = \dfrac{1}{\frac{n(n+1)}{2}} = \dfrac{2}{n(n+1)} = \dfrac{2}{n} - \dfrac{2}{n+1}$

$$S_n = \Sigma T_n = \frac{2}{1}-\frac{2}{2}+\frac{2}{2}-\frac{2}{3}........+\frac{2}{n}-\frac{2}{n+1}$$

$$S_n = 2 - \frac{2}{n+1} \quad \Rightarrow \quad S_\infty = 2$$

Match the Column :

2. (A) $\quad S_n = 4 + 11 + 22 + 37$ .... T_n

$\qquad S_n = \qquad 4 + 11 + 22 + 37$ .... T_n

$\qquad T_n = 4 + 7 + 11 + 15 +$ .... n terms

$$T_n = 4+\frac{(n-1)}{2}(14+(n-2)4) = 1+2n^2+n$$

(B) $|1^2 - 2^2 + 3^2 - 4^2 \ldots\ldots\ldots 2n \text{ terms}|$

$= |(1-2)(1+2)+(3-4)(3+4)+(5-6)(5+6)|$

$= |-3 -7 -11 -15 \ldots\ldots n \text{ terms}|$

$= 3 + 7 + 11 + \ldots\ldots n$ terms

$$=\left(\frac{n}{2}(6+(n-1)4)\right) = \frac{n}{2}(4n+2) = (2n^2+n)$$

(C) $3 + 7 + 11 + 15 \ldots\ldots\ldots = 2n^2 + n$

(D) Coefficient of x^n is

$$-2(1+2+3+\ldots\text{term}) = -\frac{2n(n+1)}{2} = -(n^2+n)$$

Assertion & Reason :

1. St.-I : $\dfrac{\frac{1}{b}+\frac{1}{a}}{\frac{2}{b}-\frac{1}{a}}+\dfrac{\frac{1}{b}+\frac{1}{c}}{\frac{2}{b}-\frac{1}{c}} = \dfrac{\frac{1}{b}+\frac{1}{a}}{\frac{1}{c}}+\dfrac{\frac{1}{b}+\frac{1}{c}}{\frac{1}{a}}$ $\left(\because \frac{2}{b}=\frac{1}{a}+\frac{1}{c}\right)$

$$= \frac{c}{b}+\frac{c}{a}+\frac{a}{b}+\frac{a}{c} = \frac{a+c}{b}+\left(\frac{c}{a}+\frac{a}{c}\right)$$

$$= \frac{(a+c)^2}{2ac}+\left(\frac{c}{a}+\frac{a}{c}\right) = \frac{1}{2}\left(\frac{a}{c}+\frac{c}{a}+2\right)+\left(\frac{c}{a}+\frac{a}{c}\right) > 4$$

[$\because$ $x + \frac{1}{x} > 2$ when $x > 0$, $x \neq \frac{1}{x}$]

St.-II is False $\therefore$ Numbers should be positive

4. St.-I : $(AM)(HM) = (GM)^2$

True for any 3 numbers in G.P.

St.-II : False if number are not in G.P.

6. $S = \ell n2 + 2\ell n2 + 5\ell n2 + \ldots\ldots\ldots T_n$ ....(i)

$S = \quad \ell n2 + 2\ell n2 + 5\ell n2 + \ldots\ldots T_n$ ....(ii)

(i) – (ii) $\Rightarrow$ $T_n = \ell n2 + \ell n2 + 3\ell n2 + \ldots\ldots$ n terms

$T_n = \ell n2 + \ell n2(n-1)^2$

now successice difference in A.P.

$\Rightarrow \quad T_n = an^2+bn+c$

St.-II is correct.

Comprehension # 2 :

$a + b + c = 5$ $(a, b, c > 0)$ $x^2y^3 = 243 = 3^5$

1. $\dfrac{a+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+c}{5} \geq \left(\dfrac{ab^3c}{27}\right)^{1/5}$ $(\because AM \geq GM)$

$\Rightarrow \quad 1^5 \geq \dfrac{ab^3c}{27} \quad \Rightarrow \quad ab^3c \leq 27$

2. Using AM $\geq$ HM

$$\frac{a+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+c}{5} \geq \frac{5}{\frac{1}{a}+\frac{3}{b}+\frac{3}{b}+\frac{3}{b}+\frac{1}{c}}$$

$$\Rightarrow \quad \frac{1}{5} \geq \frac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$$

3. Using AM $\geq$ GM

$$\frac{x^2+y+y+y+1}{5} \geq (x^2.y.y.y.1)^{1/5}$$

$$\Rightarrow \quad x^2+3y+1 \geq 5.(243)^{1/5} \geq 15$$

But x^2, $y \neq 1$, hence $x^2 + 3y + 1 > 15$

4. $\dfrac{x+x+y+y+y}{5} \geq (x^2y^3)^{1/5} \geq \dfrac{5}{\frac{2}{x}+\frac{3}{y}}$

$$\Rightarrow \quad \frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{3x+2y}$$

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. $a^x = b^y = c^z = d^u$ Taking log,

$$x\log a = y\log ar = z\log ar^2 = u\log ar^3 = \lambda$$

$(a, b, c, d \text{ in G.P.})$

$$\Rightarrow \frac{\lambda}{x} = \log a, \frac{\lambda}{y} = \log ar, \frac{\lambda}{z} = \log ar^2, \frac{\lambda}{u} = \log ar^3$$

which are in A.P. Hence x, y, z, μ in H.P.

9. $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} \ldots\ldots\ldots\ldots\ldots$

$$= \frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_1a_n}+\frac{a_2+a_{n-1}}{a_2a_{n-1}}+\frac{a_3+a_{n-2}}{a_3a_{n-2}}+\ldots.+\frac{a_1+a_n}{a_na_1}\right]$$

$$= \frac{1}{a_1+a_n}\left[\frac{1}{a_n}+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_{n-1}}\ldots\ldots+\frac{1}{a_1}+\frac{1}{a_n}\right]$$

$$= \frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\ldots\ldots+\frac{1}{a_n}\right]$$

13. $2s = a + b + c \Rightarrow a + b - c = 2s - 2c$

$$\frac{2s-2c+2s-2a+2s-2b}{3} > \left[(a+b-c)(b+c-a)(c+a-b)\right]^{1/3}$$

(Using AM > GM, $a \neq b \neq c$)

$$\Rightarrow \frac{2s}{3} > \left[(a+b-c)(b+c-a)(c+a-b)\right]^{1/3}$$

$$\Rightarrow (a+b+c) > 3\left[(a+b-c)(b+c-a)(c+a-b)\right]^{1/3}$$

$$\Rightarrow (a+b+c)^3 > 27(a+b-c)(b+c-a)(c+a-b)$$

14. $\dfrac{\frac{bc}{a}+\frac{ac}{b}}{2} \ge \left(\dfrac{abc^2}{ab}\right)^{1/2} \Rightarrow \dfrac{bc}{a}+\dfrac{ac}{b} \ge 2c$ ....(i)

Similarly $\dfrac{ac}{b}+\dfrac{ab}{c} \ge 2a$ ....(ii) (AM ≥ GM)

$\dfrac{ab}{c}+\dfrac{bc}{a} \ge 2b$ ....(iii)

Now add (i), (ii) & (iii),

$2\left(\dfrac{ab}{c}+\dfrac{bc}{a}+\dfrac{ca}{b}\right) \ge 2(a+b+c)$

15.

$$\begin{array}{c} 1 \\ 3 \quad 5 \\ 7 \quad 9 \quad 11 \\ 13 \quad 15 \quad 17 \quad 19 \end{array}$$

Total number of terms upto n^{th} row

$1 + 2 + 3 + 4 + ... + \ n = \dfrac{n(n+1)}{2}$

Total number of terms upto $(n-1)^{th}$ row

$1 + 2 + 3 + 4 + ... (n-1) = \dfrac{n(n-1)}{2}$

Sum of n^{th} row $= S_n - S_{n-1}$

$= \left(\dfrac{n(n+1)}{2}\right)^2 - \left(\dfrac{n(n-1)}{2}\right)^2$

$= \dfrac{n^2}{4}\left[(n+1)^2-(n-1)^2\right] = \dfrac{n^2}{4}4n = n^3$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

2. (a) $\dfrac{1}{1.4.7}+\dfrac{1}{4.7.10}+\dfrac{1}{7.10.13}+\ldots$

$T_n = \dfrac{1}{(3n-2)(3n+1)(3n+4)}$

$= \dfrac{1}{3n+1}\left(\dfrac{1}{3n-2}-\dfrac{1}{(3n+4)}\right)\dfrac{1}{6}$

$= \left(\dfrac{1}{(3n-2)(3n+1)}-\dfrac{1}{(3n+1)(3n+4)}\right)\dfrac{1}{6}$

$S_n = \dfrac{1}{6}\left[\dfrac{1}{1.4}-\dfrac{1}{4.7}+\dfrac{1}{4.7}-\dfrac{1}{7.10} + \dfrac{1}{(3n-2)(3n+1)}-\dfrac{1}{(3n+1)(3n+4)}\right]$

$= \dfrac{1}{6}\left[\dfrac{1}{4}-\dfrac{1}{(3n+1)(3n+4)}\right]$

now $S_\infty = \dfrac{1}{24}$

(b) $T_n = n(n+1)(n+2)(n+3)$

$= \dfrac{1}{5}[n(n+1)(n+2)(n+3)((n+4)-(n-1))]$

$= \dfrac{1}{5}[n(n+1)(n+2)(n+3)(n+4) - (n-1)(n)(n+2)(n+3)]$

$\Sigma T_n = \left(\dfrac{1}{5}\right)n(n+1)(n+2)(n+3)(n+4)$

(c) $T_n = \dfrac{1}{4n^2-1} = \dfrac{1}{2}\left[\dfrac{1}{2n-1}-\dfrac{1}{2n+1}\right]$

$S_n = \dfrac{1}{2}\left[1-\dfrac{1}{2n+1}\right]$

$S_\infty = \dfrac{1}{2}$

3. $\delta_{rs} = 0 \quad r \ne s \quad \delta_{rs} = 1$ if $r = s$

$\sum\limits_{r=1}^{n} 2^r 3^r = \sum\limits_{r=1}^{n} 6^r = 6+6^2+6^3+\ldots6^n = \dfrac{6(6^n-1)}{5}$

9. $S = \dfrac{1.3}{2}+\dfrac{3.5}{2^2}+\dfrac{5.7}{2^3}+\dfrac{7.9}{2^4}........\infty$ ..... (i)

$S\left(\dfrac{1}{2}\right) = \dfrac{1.3}{2^2}+\dfrac{3.5}{2^3}+\dfrac{5.7}{2^4}+......\infty$ ..... (ii)

(i) – (ii) gives

$\dfrac{S}{2} = \dfrac{1.3}{2}+\dfrac{3.4}{2^2}+\dfrac{4.5}{2^3}+\dfrac{4.7}{2^4}.........\infty$ ..... (iii)

$\dfrac{S}{4} = \dfrac{1.3}{2^2}+\dfrac{3.4}{2^3}+\dfrac{4.5}{2^4}.........\infty$ ..... (iv)

(iii) – (iv) gives

$\dfrac{S}{4} = \dfrac{1.3}{2}+\dfrac{3.3}{2^2}+\dfrac{4.2}{2^3}+\dfrac{4.2}{2^4}.........\infty$

$\dfrac{S}{4} = \dfrac{3}{2}+\dfrac{9}{2^2}+\dfrac{4.2}{2^3}\left(\dfrac{1}{1-\frac{1}{2}}\right)$

$\dfrac{S}{4} = \dfrac{3}{2}+\dfrac{9}{4}+2 = \dfrac{23}{4} \quad \Rightarrow \quad S = 23$

11. (a) $1-\dfrac{x}{x+1}+\dfrac{2x}{(x+1)(x+2)}+\dfrac{3x^2}{(x+1)(x+2)(x+3)}+\ldots$

$\Rightarrow \quad 1-\dfrac{x}{x+1}\left(1-\dfrac{2}{x+2}\right)+\dfrac{3x^2}{(x+1)(x+2)(x+3)}+\ldots$

$\Rightarrow \quad 1-\dfrac{x^2}{(x+1)(x+2)}+\dfrac{3x^2}{(x+1)(x+2)(x+3)}+\ldots$

$\Rightarrow \quad 1-\dfrac{x^2}{(x+1)(x+2)}\left(1-\dfrac{3}{x+3}\right)$

$$\Rightarrow \quad 1-\frac{x^3}{(x+1)(x+2)(x+3)}$$

....

....

$$\Rightarrow \quad 1-\frac{x^n}{(x+1)(x+2)(x+3)......(x+n)}$$

13. LHS = 9 + 8 10 + 8 10^2 ..... + 8 10^{n-1}

+4 10^n + 4 10^{n+1} .... + 4 10^{n+n-1}

$$= 9+8\times 10^1\left(\frac{10^{n-1}-1}{9}\right)+4\times 10^n\left[\frac{10^n-1}{9}\right]$$

$$= \frac{81+8\times 10^n-80+4\times 10^{2n}-4\times 10^n}{9}$$

$$= \frac{1+4\times 10^n+4\times 10^{2n}}{9}$$

$$= \frac{3+6\times 10^n}{9}=\left(\frac{1+2\times 10^n}{3}\right)^2$$

$$\text{RHS } = 7+6\times 10^1\,\frac{(10^{n-1}-1)}{9}=\frac{63+6\times 10^n-60}{9}$$

$$= \frac{3+6\times 10^n}{9}=\frac{1}{3}(1+2\times 10^n)$$

14. S = 1 + 5 + 13 + 29 + 61 + ... T_n

S = 1 + 5 + 13 + 29 + ...T_n

0 = 1 + 4 + 8 + 16 + 32 + ... T_n

$T_n = 1+4(2^{n-1}-1)=2^{n+1}-3$

$S_n = \Sigma(2^{n+1}-3) = (2^2 + 2^3 + ... 2^{n+1}) -3n$

$$= 2^2\left(\frac{2^n-1}{2-1}\right)-3n=2^{n+2}-4-3n$$

(b) S = 6 + 13 + 22 + 33 + ... T_n

S = 6 + 13 + 22 + ... T_n

T_n = 6 + 7 + 9 + 11 ....

= 1 + 5 + 7 + 9 + 11 ....

$= 1 + \frac{n}{2}(10+(n-1)2) = n^2 + 4n + 1$

$$S_n = \Sigma T_n=\frac{1}{6}n(n+1)(2n+13)+n$$

15.
$$\frac{\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}}{3}\ge\frac{3}{\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}}$$

$$\frac{\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}}{3}\ge\frac{3}{\frac{b}{a}+\frac{a}{b}+\frac{c}{a}+\frac{a}{c}+\frac{c}{b}+\frac{b}{c}}$$

$$\frac{\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}}{3}\ge\frac{3}{6}\qquad\left(\because\ \frac{a}{b}+\frac{b}{a}\ge 2\right)$$

$$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}\ge\frac{3}{2}$$

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

2. $\frac{a}{1-r}$ = 20 ... (i)

$\frac{a^2}{1-r^2}$ = 100 ... (ii)

from (i) and (ii)

$\frac{a}{1+r}$ = 5 ($\therefore$ a = 20(1 − r) by (i))

$\Rightarrow \frac{20(1-r)}{1+r}$ = 5

$\Rightarrow$ 5r = 3 $\Rightarrow$ r = 3/5

6. Given that A.M. = 9 and G.M. = 4

If α, β are roots of quadratic equations then quadratic equation is

$x^2 - x(\alpha + \beta) + \alpha\beta = 0$ ... (1)

A.M. = $\frac{\alpha+\beta}{2}$ = 9

$\Rightarrow \alpha + \beta = 18$ ... (2)

G.M. = $\sqrt{\alpha\beta}$ = 4

$\Rightarrow \alpha\beta = 16$ ... (3)

so the required equation will be

$x^2 - 18x + 16 = 0$

9. $\because \frac{(S_m)_{IAP}}{(S_n)_{IIAP}}=\frac{m^2}{n^2}$ then $\frac{T_m}{T_n}=\frac{2m-1}{2n-1}$

so $\frac{a_6}{a_{21}}=\frac{2\times 6-1}{2\times 21-1}=\frac{11}{41}$

12. a + ar = 12 ... (1)

$ar^2 + ar^3 = 48$ ... (2)

$ar^2(a + ar) = 48$ ... (3)

so $r^2 = 4$ a(1 + r) = 12

r = 2 a(3) = 12

because +ve G.P. $\boxed{a=4}$

13.
$$S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+.......\infty$$

$$\frac{1}{3}S=\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+......\infty$$

$$\frac{2}{3}S=1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+\frac{4}{3^5}+......\infty$$

$$=\frac{4}{3}+\frac{4/9}{1-\frac{1}{3}}=\frac{4}{3}+\frac{2}{3}=\frac{6}{3}=2\ \Rightarrow S = 3$$

14. $4500 = 150 \quad 10 + \{148 + 146 + \ldots \text{ upto n terms}\}$

$$= 1500 + \frac{n}{2}\{296 + (n-1)(-2)\}$$

$\Rightarrow n^2 - 149n + 3000 = 0$

$\Rightarrow (n - 24)(n - 125) = 0$

$\Rightarrow \quad n = 24 \qquad \because n \neq 125$

So total time taken $= 10 + 24 = 34$ min.

15. Saving after first 3 month = 600

$$600 + \left\{\frac{240 + 280 + \ldots.}{\text{let n month}}\right\} = 11040$$

$|240 + 280 + \ldots\ldots \text{ n terms}| = 10440$

$n/2\,[480 + (n-1)40] = 10440$

$n\,\{440 + 40n\} = 20880$

$n^2 + 11n - 522 = 0$

$n = 18, -29 \quad (-29 \text{ rejected})$

Total months = n + 3

18 + 3 = 21 Months

16. $\sum_{r=1}^{100} a_{2r} = a_2 + a_4 + a_6 + \ldots\ldots + a_{200} = \alpha$

$$= (a + d) + (a + 3d) + \ldots. + (a + 199d) = \alpha$$

$\sum_{r=1}^{100} a_{2r-1} = \beta = a_1 + a_3 + \ldots. + a_{199} = \beta$

$$= a + (a + 2d) + \ldots.. + (a + 198d) = \beta$$

$\frac{100}{2}[a + d + a + 199d] = \alpha$

$$\Rightarrow 50(2a + 200d) = \alpha \quad \ldots\ldots (1)$$

$\frac{100}{2}[a + a + 198d] = \beta$

$$\Rightarrow 50(2a + 198d) = \beta \quad \ldots\ldots (2)$$

(1) – (2)

$\alpha - \beta = 50(2d)$

$= d = \frac{\alpha - \beta}{100}$

17. **Statement–1 :**

$(1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3) + \ldots. + (20^3 - 19^3)$

$= 20^3 = 8000$

Statement–1 is true.

Statement–2 :

$\sum_{k=1}^{n} k^3 - (k-1)^3 = (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3)$

$+ \ldots. n^3 + (n-1)^3 = n^3.$

Statement–2 is true and Statement–2 is a correct explanation of Statement–1.

18. $100T_{100} = 50T_{50}$

$100(a + 99d) = 50(a + 4d)$

$a + 149d = 0$

$T_{150} = a + 149d = 0$

19. $S = \frac{7}{10} + \frac{77}{100} + \frac{777}{1000} + \ldots..$

$$S = \frac{7}{9}\left\{\frac{10-1}{10} + \frac{100-1}{100} + \frac{1000-1}{1000} + \ldots.\right\}$$

$$= \frac{7}{9}\left\{20 - \frac{1}{10}\left(\frac{1-10^{-20}}{9/10}\right)\right\}$$

$$= \frac{7}{9}\left\{20 - \frac{1}{9}\left(1 - 10^{-20}\right)\right\} = \frac{7}{81}(179 + 10^{-20})$$

6. $\frac{x}{1-r} = 5 \Rightarrow 5 - 5r = x \Rightarrow r = 1 - \frac{x}{5}$

As $|r| < 1$ i.e. $\left|1-\frac{x}{5}\right| < 1;\ -1 < 1 - \frac{x}{5} < 1$

$-5 < 5 - x < 5 = -10 < -x < 0 = 10 > x > 0$

i.e. $0 < x < 10$

13. $A_1 = \frac{a+b}{2}$; $G_1 = \sqrt{ab}$; $H_1 = \frac{2ab}{a+b}$

$A_n = \frac{A_{n-1}+H_{n-1}}{2}$; $G_n = \sqrt{A_{n-1}H_{n-1}}$

$H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1}+H_{n-1}}$

We know that $G^2 = AH$

So clearly $G_1 = G_2 = G_3 = \ldots.\ G_n = \sqrt{ab}$

14. A_2 is AM of A_1, H_1 and $A_1 > H_1$

$\Rightarrow A_1 > A_2 > H_1$

A_3 is A.M. of A_2, H_2

$\Rightarrow A_2 > A_3 > H_2$

$\therefore A_1 > A_2 > A_3 \ldots.$

15. as above

$A_1 > H_2 > H_1$

$A_2 > H_3 > H_2$

....

so $\ldots.. H_3 > H_2 > H_1$

$\Rightarrow H_1 < H_2 < H_3 \ldots..$

16. Let $a_1 = a,\ a_2 = ar,\ a_3 = ar^2,\ a_4 = ar^3$

Now $b_1 = a,\ b_2 = a + ar,\ b_3 = a + ar + ar^2$

$b_4 = a + ar + ar^2 + ar^3$

so b, b_2, b_3, b_4 are neither in A.P. nor in G.P. & nor in H.P.

so S(I) is true & S(II) is False.

17. $S_n = cn^2$

$S_{n-1} = c(n-1)^2$

$T_n = S_n - S_{n-1} = c\ (2n - 1)$

$T'_n = T_n^{\ 2} = c^2\ (4n^2-4n+1)$

$$\sum T'_n = nc^2\left(\frac{4n^2-1}{3}\right)$$

18. $S_k = \dfrac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$ for $k \geq 2$, $S_1 = 0$

Now $\frac{100^2}{100!} + \sum_{k=2}^{100}\left|(k^2-3k+1)\cdot\frac{1}{(k-1)!}\right| + S_1$

$\frac{100^2}{100!} + \frac{1}{1!} + \sum_{k=3}^{100}\left|\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right| + 0$

$\left(\because S_2 = \frac{1}{1!}\right)$

$= \frac{100^2}{100!} + 1 + \left|\frac{1}{0!} - \frac{1}{2!}\right| + \left|\frac{1}{1!} - \frac{1}{3!}\right| + \left|\frac{1}{2!} - \frac{1}{4!}\right| + \ldots$

$+ \left|\frac{1}{97!} - \frac{1}{99!}\right|$

$= \frac{100^2}{100!} + 3 - \frac{1}{98!} - \frac{1}{99!}$

$= \frac{100^2}{100!} + 3 - \frac{100}{99!} = 3$

19. $a_1 = 15$

$27 - 2a_2 > 0$

$a_k = 2a_{k-1} - a_{k-2}\ \forall\ k = 3, 4, 5 \ldots.\ 11$

$\frac{a_1^2 + a_2^2 + \ldots\ldots.. a_{11}^2}{11} = 90$

$a_{k-1} = \frac{a_k + a_{k-2}}{2}$

all $a_i(i = 1, 2, \ldots\ldots.\ 11)$ are in A.P.

Let the numbers are

$(a_6+5d), (a_6+4d), \ldots\ldots, a_6, \ldots., (a_6 - 4d), (a - 5d)$

$11a_6^2 + 110d^2 = 990$

$a_6 = 15 - 5d$

$a_6^2 + 10d^2 = 90$

$(15 - 5d)^2 + 10d^2 = 90 \Rightarrow 7d^2 - 30d + 27 = 0$

$\Rightarrow d = 3, \frac{9}{7}$

for $d = 3 \Rightarrow a_2 = 12$ (possible)

for $d = 9/7 \Rightarrow a_2 = 13.7$ (not possible since $a_2 < 13.5$)

$a_6 = 0 \Rightarrow \frac{a_1 + a_2 + \ldots + a_{11}}{11} = a_6 = 0$

20. As $a > 0$

and all the given terms are positive

hence considering A.M. $\geq$ G.M. for given numbers :

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^{8} + a^{10}}{7}$$

$$\geq \left(a^{-5}.a^{-4}.a^{-3}.a^{-3}.a^{-3}.a^{8}.a^{10}\right)^{\frac{1}{7}}$$

$\Rightarrow \quad \dfrac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+a^{8}+a^{10}}{7} \geq 1$

$\Rightarrow \quad \left(a^{-5}+a^{-4}+3a^{-3}+a^{8}+a^{10}\right)_{min} = 7$

where $a^{-5} = a^{-4} = a^{-3} = a^{8} = a^{10}$ i.e. $a = 1$

$\Rightarrow \quad (a^{-5} + a^{-4} + 3a^{-3} + a^{8} + a^{10} + 1)_{min} = 8$ when $a = 1$

21. Consider $d \neq 0$ the solution is

$a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_{100} \rightarrow AP$

$a_1 = 3 \quad ; \quad S_p = \sum_{i=1}^{p} a_i \qquad 1 \leq n \leq 20$

$m = 5n$

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a_1+(m-1)d]}{\frac{n}{2}[2a_1+(n-1)d]}$$

$$\frac{S_m}{S_n} = \frac{5[(2a_1-d)+5nd]}{[(2a_1-d)+nd]}$$

for $\dfrac{S_m}{S_n}$ to be independent of n

$\therefore \quad 2a_1 - d = 0 \quad \Rightarrow \quad d = 2a_1 \Rightarrow \quad d = 6$

$\Rightarrow \quad a_2 = 9$

If $d = 0 \quad \Rightarrow \quad a_2 = a_1 = 3$

22. $a_1, a_2, a_3 \ldots\ldots\ldots$ be in H.P

$\Rightarrow \dfrac{1}{a_1}, \dfrac{1}{a_2}, \dfrac{1}{a_3} \ldots.$ be in A.P.

in A.P. $T_1 = \dfrac{1}{a_1} = \dfrac{1}{5}$ and $T_{20} = \dfrac{1}{a_{20}} = \dfrac{1}{25}$

$\therefore \; T_{20} = T_1 + 19d$

$\dfrac{1}{25} = \dfrac{1}{5} + 19d \Rightarrow d = -\dfrac{4}{19 \times 25}$

$T_n = T_1 + (n - 1)d < 0$

$\Rightarrow \dfrac{1}{5} - \dfrac{(n-1).4}{19 \times 25} < 0 \Rightarrow \dfrac{1}{5} < \dfrac{4(n-1)}{25 \times 19}$

$\Rightarrow \dfrac{5 \times 19}{4} + 1 < n \quad \Rightarrow \dfrac{99}{4} < n$

$\Rightarrow$ least positive integer n is 25.

23. $S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 \ldots\ldots.$

$S_n = (3^2 - 1^2) + (4^2 - 2^2) + \ldots\ldots\ldots\ldots$

$S_n = 2(1 + 2 + 3 + \ldots\ldots.. + 4n)$

$= \dfrac{2(4n)(4n+1)}{2}$

$S_n = 4n\left(4n+1\right)$

$S_n = 4n(4n + 1) = 1056$ is possible when $n = 8$

$4n\,(4n + 1) = 1088$ not possible

$4n(4n + 1) = 1120$ not possible

$4n(4n + 1) = 1332$ possible when $n = 9$.

24. When 1 and 2 are removed from numbers 1 to n then we get maximum possible sum of remaining numbers and when $n-1, n$ are removed then we get minimum possible sum of remaining numbers.

$\Rightarrow \quad \dfrac{n(n+1)}{2} - \left(2n-1\right) \leq 1224 \leq \dfrac{n\left(n+1\right)}{2} - 3$

$\Rightarrow \quad \begin{cases} n^2 + n - 2454 \geq 0 \\ n^2 - 3n - 2446 \leq 0 \end{cases}$

$\Rightarrow \quad \begin{cases} n \geq 50 \\ n \leq 50 \end{cases} \Rightarrow n = 50$

Now let x and x + 1 be two consecutive numbers

$\Rightarrow \quad \dfrac{50\left(50+1\right)}{2} - x - x - 1 = 1224$

$\Rightarrow \quad x = 25$

$\Rightarrow \quad 25^{th}$ and 26^{th} cards are removed from pack

$\Rightarrow \quad k = 25 \Rightarrow k - 20 = 5$