POINT & STRAIGHT LINE

EXERCISE - 01 CHECK YOUR GRASP

7. Let (h, k) be the centroid of triangle

$3h = \cos\alpha + \sin\alpha + 1$

$\Rightarrow (3h - 1) = \cos\alpha + \sin\alpha$ ....(1)

$3k = \sin\alpha - \cos\alpha + 2$

$\Rightarrow (3k - 2) = \sin\alpha - \cos\alpha$ ....(2)

square & add (1) & (2)

$9(x^2 + y^2) + 6(x - 2y) = -3$

8. $\Delta = \frac{1}{2}\begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0$

$\Rightarrow (2a - c)(2b - c) - (3a - c)(3b - c) = 0$

$\Rightarrow 4ab - 2ac - 2bc + c^2 - (9ab - 3ac - 3bc + c^2) = 0$

$\Rightarrow ac + bc - 5ab = 0$

$\frac{1}{a} + \frac{1}{b} = \frac{5}{c} \Rightarrow \frac{1}{a} + \frac{1}{b} = 2\left(\frac{5}{2c}\right)$

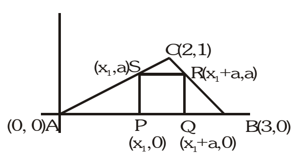
$\therefore$ a, $\frac{2c}{5}$, b are in H.P.

14. Let co-ordinates of P are $(x_1, 0)$ and side of square is 'a'

$\therefore$ Q $(x_1 + a, 0)$

S (x_1, a)

R $(x_1 + a, a)$



Now,

$m_{AS} = m_{AC}$

$\Rightarrow \frac{a}{x_1} = \frac{1}{2} \Rightarrow x_1 = 2a$ ....(1)

$m_{BR} = m_{BC}$

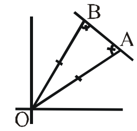
$\Rightarrow \frac{a}{x_1 + a - 3} = -1 \Rightarrow x_1 + 2a - 3 = 0$ ...(2)

from (1) & (2) $a = \frac{3}{4}$ & $x_1 = \frac{3}{2}$

co-ordinates of P, Q, R, & S can be determined.

17. $m_{OA} = 1$, $m_{OB} = 7$

Let $m_{AB} = m$



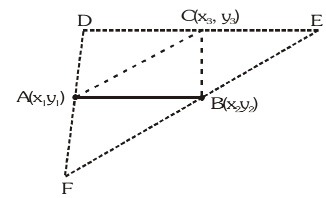
AO = OB

$\Rightarrow \angle OBA = \angle OAB$

$\frac{m-7}{1+7m} = \frac{1-m}{1+m} \Rightarrow m = -\frac{1}{2}, 2$

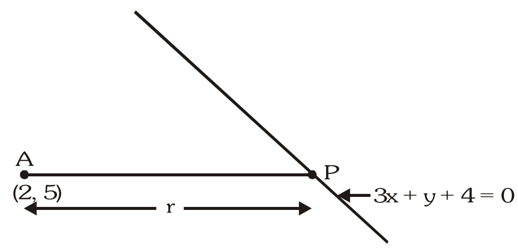
but slope of AB is negative $\therefore m = -\frac{1}{2}$

24.



ABCD, ABEC, ACBF are three possible parallelograms.

29. Let distance be 'r'.



Co-ordinates of 'P' are

$(2 + r\cos\theta, 5 + r\sin\theta)$ where $\tan\theta = \frac{3}{4}$

which lies on the line $3x + y + 4 = 0$

$3(2 + r\cos\theta) + 5 + r\sin\theta + 4 = 0$

$r\left(3.\frac{4}{5} + \frac{3}{5}\right) + 15 = 0 \Rightarrow r = -\frac{15}{3} = -5$

but distance can not be negative

$\therefore r = 5$

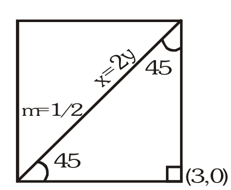
31. Here, $x + 2y - 3 = 0$ and $3x + 4y - 7 = 0$ intersect (1, 1), which does not satisfy $2x + 3y - 4 = 0$ and $4x + 5y - 6 = 0$. Also, $3x + 4y - 7 = 0$ and $2x + 3y - 4 = 0$ intersect at (5, –2) which does not satisfy $x + 2y - 3 = 0$ and $4x + 5y - 6 = 0$

Intersection point of $x + 2y - 3 = 0$ and $2x + 3y - 4 = 0$ is (–1, 2) which satisfy $4x + 5y - 6 = 0$.

Hence, only three lines are concurrent.

34. Let slope of required line is m

$\tan 45° = \left|\frac{\frac{1}{2} - m}{1 + \frac{m}{2}}\right|$



$\Rightarrow \pm 1 = \frac{\frac{1}{2} - m}{1 + \frac{m}{2}} \Rightarrow m = -\frac{1}{3}, 3$

equation of lines $y = 3(x - 3)$ & $y = -\frac{1}{3}(x-3)$

35. $p(x + y - 1) + q(2x - 3y + 1) = 0$

$x + y - 1 + \frac{q}{p}(2x - 3y + 1) = 0$

$L_1 + \lambda L_2 = 0$

$\therefore$ line always passes through point of intersection of $L_1 = 0$ & $L_2 = 0$.

38. Homogenizing the curve with the help of the straight line.

$5x^2+12xy-6y^2+4x(x+ky)-2y(x+ky)+3(x+ky)^2 = 0$

$12x^2 + (10 + 4k + 6k)xy + (3k^2 - 2k - 6)y^2 = 0$

Lines are equally inclined to the coordinate axes

$\therefore$ coefficient of $xy = 0$

$\Rightarrow 10k + 10 = 0 \quad \Rightarrow k = -1$

43. As, $3x + 2y \ge 0$ ....(1)

where (1, 3)(5, 0) and (–1, 2) satisfy (1)

again, $2x + y - 13 \ge 0$

is not satisfied by (1, 3), $\therefore$ (b) is false

$2x - 3y - 12 \ge 0$,

is satisfied for all points, $\therefore$ (c) is true

$-2x + y \ge 0$,

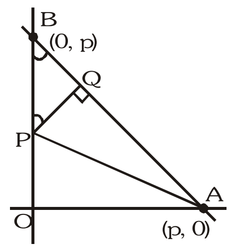
is not satisfied by (5, 0), $\therefore$ (d) is false.

Thus (a), (c) are correct answers.

EXERCISE - 02 **BRAIN TEASERS**

2. $\angle ABO = \pi/4 = \angle BPQ$

($\therefore$ PQ = BQ)



$AB^2 = OA^2 + OB^2$

$(AQ + BQ)^2 = 2p^2$ ... (1)

$Ar(\Delta APQ) = \frac{3}{8} Ar(\Delta OAB)$

$\frac{1}{2} PQ \; AQ = \frac{3}{8} \cdot \frac{1}{2} \cdot p^2$

$BQ \cdot AQ = \frac{3}{8} p^2$ ... (2)

From (1) & (2)

$(AQ + BQ)^2 = 2 \cdot \frac{8}{3} AQ \cdot BQ$

$\Rightarrow AQ + BQ = \frac{4}{\sqrt{3}} \sqrt{AQ.BQ}$

$\Rightarrow \sqrt{\frac{AQ}{BQ}} + \sqrt{\frac{BQ}{AQ}} = \frac{4}{\sqrt{3}}$

$\Rightarrow t + \frac{1}{t} = \frac{4}{\sqrt{3}}$ (Let $\sqrt{\frac{AQ}{BQ}} = t$)

$\Rightarrow \sqrt{3}t^2 + \sqrt{3} = 4t \Rightarrow \sqrt{3}t^2 - 4t + \sqrt{3} = 0$

$\therefore \quad t = \sqrt{3}$ or $\frac{1}{\sqrt{3}}$

$\therefore \frac{AQ}{BQ} = 3$ or $\frac{1}{3}$

Now, if $\frac{AQ}{BQ} = \frac{1}{3}$ then coordinates of Q are $\left(\frac{3p}{4}, \frac{p}{4}\right)$ and equation of line PQ is

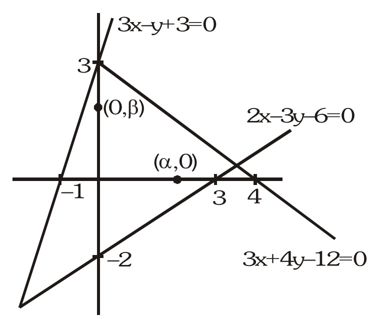
$y - \frac{p}{4} = 1\left(x - \frac{3p}{4}\right)$

putting $x = 0$, $y = -\frac{p}{2}$

$\therefore$ P lies on negative y-axis ($\therefore \frac{AQ}{BQ} = \frac{1}{3}$ is rejected)

$\therefore \frac{AQ}{BQ} = 3$

4.



(i) $4\beta - 12 < 0 \Rightarrow \beta < 3$

$3\alpha - 12 < 0 \Rightarrow \alpha < 4$

(ii) $-\beta + 3 > 0 \Rightarrow \beta < 3$

$3\alpha + 3 > 0 \Rightarrow \alpha > -1$

(iii) $2\alpha - 6 < 0 \Rightarrow \alpha < 3$

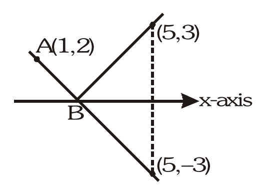
$-3\beta - 6 < 0 \Rightarrow \beta < -2$

From cases (i), (ii) & (iii)

$\alpha \in [-1, 3]$ & $\beta \in [-2, 3]$

6. Image of (5, 3) in x-axis is (5, – 3)

Now, line AB will also pass through (5, – 3)



$\therefore$ equation of AB is $y - 2 = \frac{-5}{4}(x - 1)$

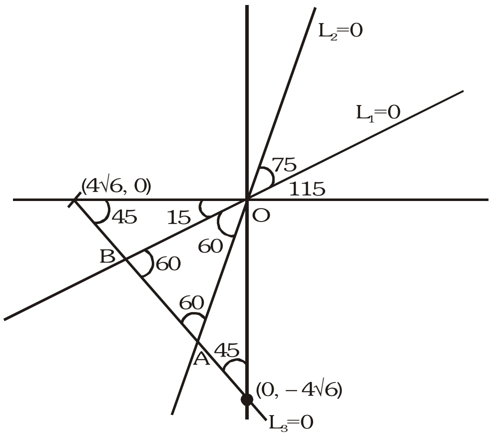
$5x + 4y = 13$

9. $x^2 - 4xy + y^2 = 0$

$y^2 - 4xy + 4x^2 = 3x^2$

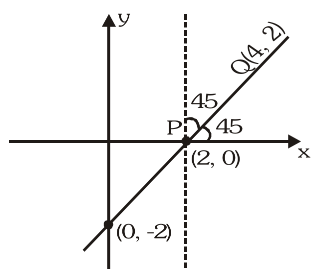
$(y - 2x)^2 = 3x^2$

$y = (2 \pm \sqrt{3})x$



$\therefore$ Δ formed by $L_1 = 0$, $L_2 = 0$ & $L_3 = 0$ is equilateral

11. $m_{PQ} = \dfrac{2-0}{4-2} = 1$



Now, PQ is rotated through 45 about P in anticlockwise direction

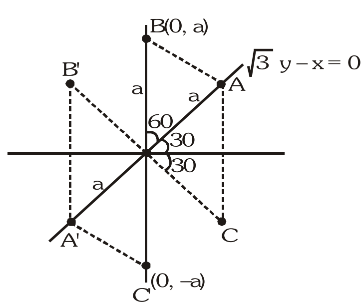
$\therefore$ slope of PQ becomes infinity.

$\therefore$ Equation of PQ in new position is $x = 2$

16. Possible points on the line $\sqrt{3}\,y - x = 0$ as second vertex of the equilateral triangle are A and A' correspondingly possible coordinates of the 3rd vertex are B(0, a), C(a cos 30 , – a sin 30),

B' (–a cos 30 , a sin 30), C' (0, – a)

$\equiv B(0,\ a), C\left(\dfrac{a\sqrt{3}}{2}, -\dfrac{a}{2}\right), B'\left(\dfrac{-a\sqrt{3}}{2}, \dfrac{a}{2}\right), C\ (0,\ -a)$



18. (i) After reflection about the line $y = x$, (4, 1) becomes (1, 4)

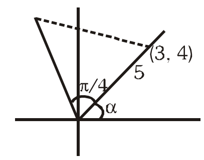
(ii) After 2nd transformation it becomes (3, 4)

(iii) In the 3rd transformation point has been rotated in anticlockwise at an angle of $\pi/4$ about origin

$\therefore$ its coordinate becomes

$(5\cos(\pi/4 + \alpha),\ 5\sin(\pi/4 + \alpha))$

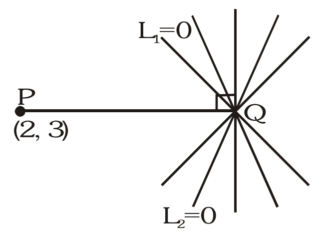
where $\tan \alpha = \dfrac{4}{3}$



Solving above we can set the coordinates of final position of the point.

20. $L_1 : 3x + 4y + 6 = 0$

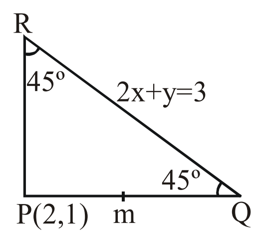
$L_2 : x + y + 2 = 0$



Line situated at greatest distance from P(2, 3) is the line passing through point of intersection (Q) of the given lines and perpendicular to PQ.

22. Let m be the slope of PQ, then

$\tan 45 = \left|\dfrac{m-(-2)}{1+m(-2)}\right|$



$\Rightarrow 1 = \left|\dfrac{m+2}{1-2m}\right| \qquad \Rightarrow \pm 1 = \dfrac{m+2}{1-2m}$

$\Rightarrow m = -1/3$ or $m = 3$

As PR also makes $\angle 45$ with RQ . $\therefore$ The above two values of m are for PQ and PR.

$\therefore$ Equation of PQ $\quad y - 1 = -\dfrac{1}{3}(x - 2)$

$\Rightarrow x + 3y - 5 = 0$

and equation of PR is $\Rightarrow 3x - y - 5 = 0$

$\therefore$ Combined equation of PQ and PR is

$(x - 3y - 5)(3x - y - 5) = 0$

$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

Match the column :

2. (A) Let the lines $4x + 5y = 0$ and $7x + 2y = 0$ represents the sides AB & AD of the parallelogram ABCD, then the vertices of A, B, D are (0,0), $\left(\frac{5}{3}, -\frac{4}{3}\right)$ and $\left(-\frac{2}{3}, \frac{7}{3}\right)$ respectively the mid point of BD is $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ the equation of the line passing through $\left(\frac{1}{2}, \frac{1}{2}\right)$ and (0, 0) will be $x - y = 0$ which is the required equation of the other diagonal

So $a = 1, b = -1, c = 0$

$\therefore$ $a + b + c = 0$

(B) Joint equation of lines OA & OB, O being the origin will be

$2x^2 - by^2 + (2b - 1)xy - (x + by)(-2x+by) = 0$

$\Rightarrow$ $4x^2 - (b + b^2)y^2 + (3b - 1)xy = 0$

If these lines are perpendicular then

$4 - b - b^2 = 0 \Rightarrow b + b^2 = 4$

(C) Equation of line passing through intersection of $4x + 3y = 12$ and $3x + 4y = 12$ will be

$(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$

If passes through (3, 4) $\Rightarrow$ $(12 + \lambda(13)) = 0$

$\Rightarrow$ $\lambda = -\frac{12}{13}$

$\therefore$ Equation of the required line

$16x - 9y - 12 = 0$

length of intercepts on x and y axes are $\frac{3}{4}$ and $\frac{4}{3}$ so $ab = 1$

Comprehension # 2

1. d(OR) = d(AR)

$|x - 0| + |y - 0| = |x - 1| + |y - 2|$

$x + y = |x - 1| + |y - 2|$ $(\because x > 0, y > 0)$

$x + y = -x + 1 - y + 2$

$(\because 0 \le x < 1 \ \& \ 0 \le y < 2)$

$2x + 2y = 3.$

2. d(OS) = d(BS)

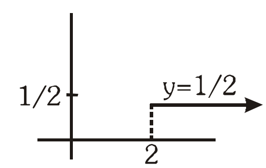
$|x - 0| + |y - 0| = |x - 2| + |y - 3|$

$x + y = x - 2 + 3 - y$

$(\because x \ge 2 \ \& \ 0 \le y < 3).$

$y = 1/2.$

which is an infinite ray



3. d(TO) = d(TC)

$|x - 0| + |y - 0| = |x - 4| + |y - 3|$

$x + y = |x - 4| + |y - 3|$

Case:I $0 \le x < 4 \ \& \ 0 \le y < 3.$

$x + y = -x + 4 - y + 3$

$x + y = 7/2.$

Case:II $0 \le x < 4 \ \& \ y \ge 3.$

$x + y = -x + 4 + y - 3$

$x = 1/2.$

Case:III $x \ge 4 \ \& \ 0 \le y < 3.$

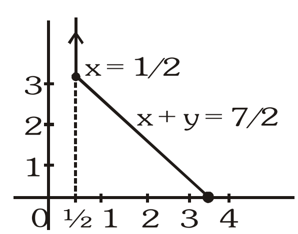
$x + y = x - 4 - y + 3$

$y = -1/2.$

Case:IV $x \ge 4 \ \& \ y \ge 3.$

$x + y = x - 4 + y - 3$

$0 = -7$ (so rejected)



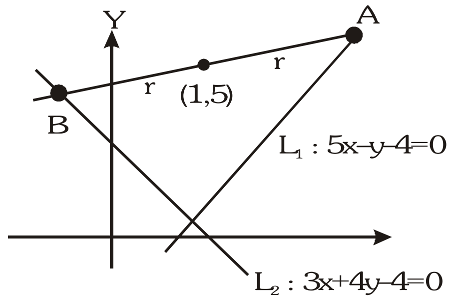
5. Coordinate of A which is at a distance 'r' from (1, 5) is $(1 + r\cos\theta, 5 + r\sin\theta)$

which satisfies $L_1 = 0$

$5(1 + r\cos\theta) - (5 + r\sin\theta) - 4 = 0$

$r(5\cos\theta - \sin\theta) = 4$

$$r = \frac{4}{5\cos\theta - \sin\theta}$$



Similarly coordinate of point B $(1-r\cos\theta, 5-r\sin\theta)$ which lies on $L_2 = 0$

$\therefore 3(1 - r\cos\theta) + 4(5 - r\sin\theta) - 4 = 0$

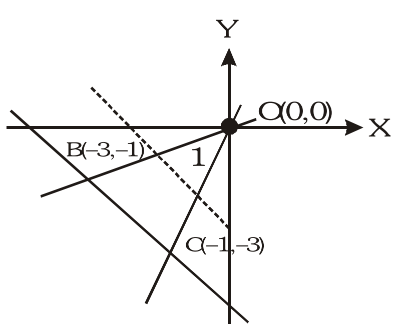
$$r = \frac{19}{3\cos\theta + 4\sin\theta}$$

Equating both values of r & get $\tan\theta$ (= m) and equation of the line passing through (1, 5).

8. Equation of BC is $x + y + 4 = 0$

Equation of the line parallel to BC is

$x + y + \lambda = 0$



and its perpendicular distance from the origin is 1/2.

$$\therefore \left|\frac{\lambda}{\sqrt{2}}\right| = \frac{1}{2} \Rightarrow \lambda = \pm\frac{1}{\sqrt{2}}$$

but x & y-intercept is negative $\therefore \lambda = \frac{1}{\sqrt{2}}$

Hence equation is $2x + 2y + \sqrt{2} = 0$

11. Consider a line $\ell x + my + n = 0$

point $\left(\frac{r^3}{r-1}, \frac{r^2-3}{r-1}\right)$ lies on the above line

$$\therefore \ell\left(\frac{r^3}{r-1}\right) + m\left(\frac{r^2-3}{r-1}\right) + n = 0$$

$\ell r^3 + mr^2 + nr - (3m + n) = 0$

a, b, c are the roots of the equation.

$$a+b+c = \frac{-m}{\ell}, \quad ab+bc+ca = \frac{n}{\ell}, \quad abc = \frac{3m+n}{\ell}$$

Now taking LHS

$$3(a + b + c) = \frac{-3m}{\ell}$$

RHS

$$ab + bc + ca - abc = \frac{n}{\ell} - \left(\frac{3m+n}{\ell}\right) = -\frac{3m}{\ell}$$

13. Let point of intersection of lines is (x, y)

using parametric form of line

$$\frac{x-2}{\cos\theta} = \frac{y-1}{\sin\theta} = 3$$

$x = 3\cos\theta + 2, \quad y = 3\sin\theta + 1$

This point satisfy equation of line

$$4y - 4x + 4 + 3\sqrt{2} + 3\sqrt{10} = 0$$

$$12(\sin\theta - \cos\theta) = -3\sqrt{2}(1+\sqrt{5})$$

$$\sin\theta - \cos\theta = -\frac{(1+\sqrt{5})}{2\sqrt{2}}$$

$$\Rightarrow \cos(\theta + 45°) = -\frac{(1+\sqrt{5})}{4} \quad ....(1)$$

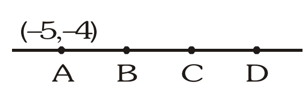
$$\Rightarrow \cos(\theta + 45°) = \cos(180° - 36°)$$

$$\Rightarrow \cos(\theta + 45°) = \cos 144° \Rightarrow \theta = 99°$$

Now from (1)

$$\cos(\theta + 45°) = \cos(180° + 36°) \Rightarrow \theta = 171$$

16.



$AB = r_1 \quad AC = r_2 \quad AD = r_3$

Using parametric form of line, we get

$B \equiv (-5 + r_1\cos\theta, -4 + r_1\sin\theta)$

it lie on line $x + 3y + 2 = 0$

$$AB = r_1 = \left|\frac{-5-12+2}{\cos\theta + 3\sin\theta}\right| = \frac{15}{\cos\theta + 3\sin\theta}$$

Similarly $r_2 = \frac{10}{2\cos\theta + \sin\theta}$ & $r_3 = \frac{6}{\cos\theta - \sin\theta}$

Now $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$

$(\cos\theta + 3\sin\theta)^2 + (2\cos\theta + \sin\theta)^2 = (\cos\theta - \sin\theta)^2$

$\Rightarrow \quad 4\cos^2\theta + 9\sin^2\theta + 12\sin\theta\cos\theta = 0$

$\Rightarrow \quad (2\cos\theta + 3\sin\theta)^3 = 0$

$\Rightarrow \quad \tan\theta = -\frac{2}{3}$

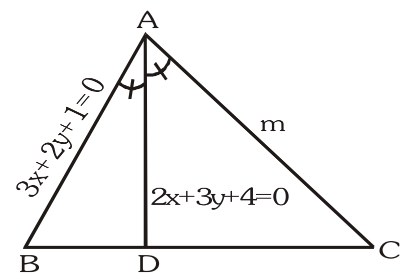
equation of line through A is $(y + 4) = -\frac{2}{3}(x+5)$

$\Rightarrow \quad 2x + 3y + 22 = 0$

18. $\because \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow$ AD is bisector of $\angle A$.

Let slope of AC be 'm'



$\therefore \left|\frac{m - \left(\frac{-2}{3}\right)}{1 + m\left(\frac{-2}{3}\right)}\right| = \left|\frac{-\frac{2}{3} - \left(-\frac{3}{2}\right)}{1 + \left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right)}\right|$

Solving above we get $m = \frac{-9}{46}, \frac{-3}{2}$ (slope of AB)

$\therefore$ Equation of AC can be obtained taking slope $= \frac{-9}{46}$

19. Let the equation of the chord be $y = mx + c$

Now homogenizing the curve

$3x^2 + 3y^2 - 2x\left(\frac{y - mx}{c}\right) + 4y\left(\frac{y - mx}{c}\right) = 0$

$(3c + 2m)x^2 + (3c + 4)y^2 - (2 + 4m)xy = 0$

coefficient of x^2 + coefficient of $y^2 = 0$

$3c + 2m + 3c + 4 = 0$

$\Rightarrow 3c + m + 2 = 0$

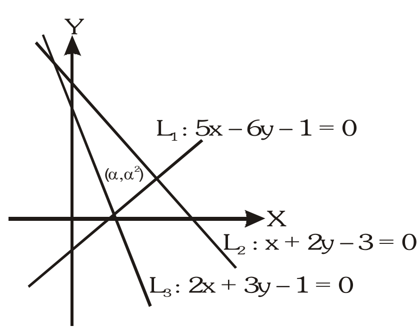
$\Rightarrow \frac{m}{3} + \frac{2}{3} + c = 0$ ... (1)

$\Rightarrow mx - y + c = 0$ ... (2)

$\therefore$ chord (2) always passes through $\left(\frac{1}{3}, -\frac{2}{3}\right)$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

2. Point (α, α^2) and origin lie on same side of $L_1 = 0$ & $L_2 = 0$ and opposite side of $L_3 = 0$.



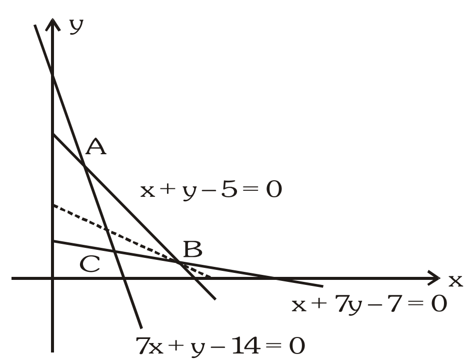
$\therefore 5\alpha - 6\alpha^2 - 1 < 0$ ... (1)

$\alpha + 2\alpha^2 - 3 < 0$ ... (2)

$2\alpha + 3\alpha^2 - 1 > 0$ ... (3)

Solving (1), (2) and (3) we can get the values of 'α'.

8. Interior bisector of $\angle$ B is the non origin containing bisector between AB and BC.



$\therefore \frac{-x - y + 5}{\sqrt{2}} = -\frac{-x - 7y + 7}{5\sqrt{2}}$

$-5x - 5y + 25 = x + 7y - 7$

$6x + 12y - 32 = 0$

$3x + 6y - 16 = 0$

Exterior bisector of $\angle C$ is also the non origin containing bisector between AC and BC

$\frac{-7x - y + 14}{5\sqrt{2}} = -\frac{-x - 7y + 7}{5\sqrt{2}}$

$8x + 8y - 21 = 0$

Now,

$AB : -x - y + 5 = 0$

$AC : -7x - y + 14 = 0$

Now, $a_1a_2 + b_1b_2 = 7 + 1 = 8 > 0$

$\therefore$ origin lies in obtuse region and interior bisector of $\angle A$ is non-origin containing.

$\therefore$ It is an acute angle.

10. $y + 2at = tx - at^3$

slope = t.

Let is passes through P(h, k)

$\therefore k + 2at = th - at^3$

$at^3 + t(2a - h) + k = 0$ ... (1)

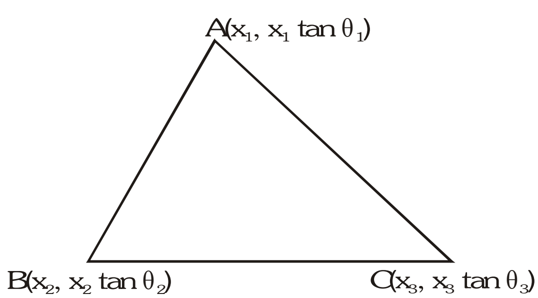
$t_1t_2t_3 = -\frac{k}{a}$ $\{t_1t_2 = -1\}$

$t_3 = \frac{k}{a}$

Substituting t_3 in (1) we can get the desired locus

13. Circumcentre is origin

$\therefore OA^2 = OB^2 = OC^2$



$x_1^2 + x_1^2 \tan^2\theta_1 = x_2^2 + x_2^2 \tan^2\theta_2$

$= x_3^2 + x_3^2 \tan^2\theta_3 = r^2$

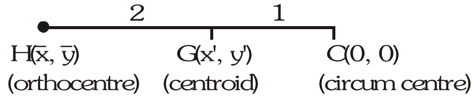
$x_1 = r\cos\theta_1,\ x_2 = r\cos\theta_2,\ x_3 = r\cos\theta_3$

$\therefore$ co-ordinate of vertices of the triangle become -

$A(r\cos\theta_1, r\sin\theta_1)$, $B(r\cos\theta_2, r\sin\theta_2)$,

$C(r\cos\theta_3, r\sin\theta_3)$

$x' = \frac{\Sigma r\cos\theta_1}{3}$, $y' = \frac{\Sigma r\sin\theta_1}{3}$



Now, $x' = \frac{0 + \bar{x}}{3}$

$\bar{x} = r(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)$

$\bar{y} = r(\sin\theta_1 + \sin\theta_2 + \sin\theta_3)$

$\therefore \frac{\bar{x}}{\bar{y}} = \frac{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}$

14. Let OA = a and OB = b. Then, the co-ordinates of A and B are (a, 0) and (0, b) respectively.

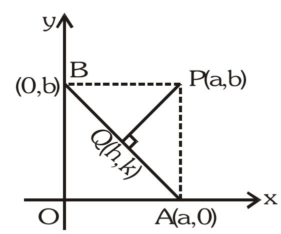
The co-ordinates of P are (a, b). Let θ be the foot of perpendicular from P on AB and let the co-ordinates of Q(h, k). Here a and b are the variable and we have to find locus of Q.

Now, AB = c

$\Rightarrow AB^2 = c^2 \Rightarrow OA^2 + OB^2 = c^2$

$\Rightarrow a^2 + b^2 = c^2$ ....(1)

Now, $PQ \perp AB$



$\Rightarrow$ slope of AB. slope of PQ = –1

$\Rightarrow \frac{k-b}{h-a}\cdot\frac{0-b}{a-0} = -1$

$\Rightarrow bk - b^2 = ah - a^2$

$\Rightarrow ah - bk = a^2 - b^2$ ....(2)

Equation of line AB is $\frac{x}{a} + \frac{y}{b} = 1$

Since, Q lies on AB,

$\frac{h}{a} + \frac{k}{b} = 1 \Rightarrow bh + ak = ab$ ....(3)

Solving (2) and (3), we get

$$\frac{h}{ab^2 + a(a^2 - b^2)} = \frac{k}{-b(a^2 - b^2) + a^2b} = \frac{1}{a^2 + b^2}$$

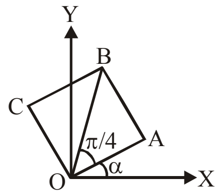
$\Rightarrow \frac{h}{a^3} = \frac{k}{b^3} = \frac{1}{c^2}$ {using (1)}

$\Rightarrow a = (hc^2)^{1/3}$ and $b = (kc^2)^{1/3}$

Substituting the values of a and b in $a^2 + b^2 + c^2$, we get $h^{2/3} + k^{2/3} = c^{2/3}$

or $x^{2/3} + y^{2/3} = c^{2/3}$ required locus.

3. Co-ordinates of A = $(a\cos\alpha, a\sin\alpha)$



Equation of OB, $y = \tan\left(\frac{\pi}{4}+\alpha\right)x$

$\because$ CA $\perp^r$ to OB

$\therefore$ Slope of CA = $-\cot\left(\frac{\pi}{4}+\alpha\right)$

Equation of CA,

$y - a\sin\alpha = -\cot\left(\frac{\pi}{4}+\alpha\right)(x - a\cos\alpha)$

$\Rightarrow$ $y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a$

$\Rightarrow$ $y(\cos\alpha + \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

7. Here a + b = 1. Required line is

$\frac{x}{a} - \frac{y}{1+a} = 1$ ..... (i)

Since line (i) passes through (4, 3)

$\therefore \frac{4}{a} - \frac{3}{1+a} = 1 \Rightarrow 4 + 4a - 3a = a + a^2$

$\Rightarrow a^2 = 4 \Rightarrow a = \pm 2$

$\therefore$ Required lines are $\frac{x}{2} - \frac{y}{3} = 1$ & $\frac{x}{-2} + \frac{y}{1} = 1$

10. The lines passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow$ $(a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$ ..... (i)

Line (i) is parallel to x-axis

$\therefore a + b\lambda = 0 \Rightarrow \lambda = \frac{-a}{b} = 0$

Put the value of λ in (i)

$ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$

$y\left(2b+\frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0,$

$y\left(\frac{2b^2+2a^2}{b}\right) = -\left(\frac{3b^2+3a^2}{b}\right)$

$y = \frac{-3(a^2+b^2)}{2(b^2+a^2)} = \frac{-3}{2}, y = -\frac{3}{2}$

So, it is 3/2 unit below x-axis

11. a, b, c are in H.P., then $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ ..... (i)

Given line is $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ ..... (ii)

Subtracting both $\frac{1}{a}(x - 1) + \frac{1}{b}(y + 2) = 0$

Since $a \neq 0,\ b \neq 0$

So, $(x - 1) = 0 \Rightarrow x = 1$ and $(y + 2) = 0 \Rightarrow y = -2$

Trick : Checking from options, let a, b, c are $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}$

Then x + 2y + 3 = 0 will satisfy (3) option

17. Lines are ||

$\frac{-p(p^2+1)}{-1} = \frac{-(p^2+1)^2}{p^2+1}$

$p(p^2 + 1) = -(p^2 + 1)$

$p = -1$

20. **Case–I**

Let a > 0

x + y = a and ax – y = 1

$\therefore$ x = 1   y = a – 1

$\therefore$ a – 1 > 0

$\Rightarrow$ a > 1

Case–II

Let a < 0

x + y = –a and –a – y = 1

But line x + y = –a does not passes through the I[st] quadrant.

Case–III

a = 0

x + y = 0 .... (I)

y = –1 .... (II)

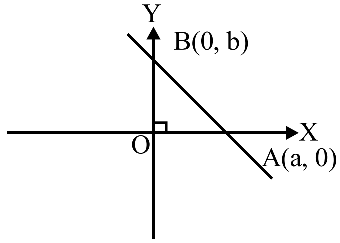
line (I) and line (II) do not pass through I[st] quadrant.

Hence $a \in (1, \infty)$

21. Let the equationof line be $\frac{x}{a}+\frac{y}{b}=1$

It passes through (1, 2)

$\therefore \quad \frac{1}{a}+\frac{2}{b}=1$



$\Rightarrow \; b = \frac{2a}{a-1}$

Area of $\Delta = \frac{1}{2}ab$

$\Rightarrow \; \Delta = \frac{a^2}{a-1}$

$\frac{d\Delta}{da} = \frac{a^2-2a}{(a-1)^2}=0$

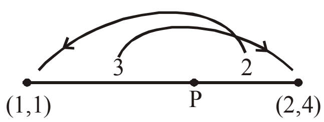
$\Rightarrow \; \frac{a(a-2)}{(a-1)^2}=0 \Rightarrow a = 0, 2$

but at a = 0, Δ not possible

$\therefore$ a = 2

slope of line $-\frac{b}{2}$ = 2

22. Let P be the point dividing (1, 1) and (2, 4) in the ratio 3 : 2.



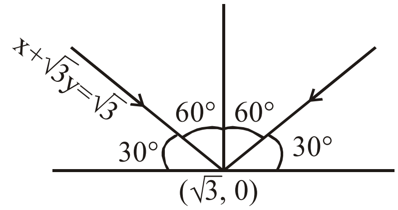
$\therefore \quad$ P is $\left(\frac{8}{5}, \frac{14}{5}\right)$.

Put $x = \frac{8}{5}$ and $y = \frac{14}{5}$ in 2x + y = k

$2 \times \frac{8}{5} + \frac{14}{5} = k$

$\Rightarrow \quad$ k = 6

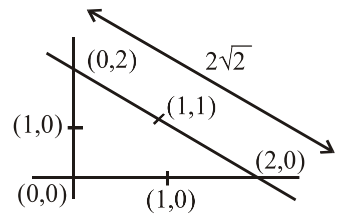
23.



$y = \frac{1}{\sqrt{3}}(x-\sqrt{3})$

$\sqrt{3}y = x-\sqrt{3}$

24.



$x = \frac{ax_1+bx_2+cx_3}{a+b+c}$

$= \frac{2(0)+2\sqrt{2}(0)+2(2)}{2+2+2\sqrt{2}}$

$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$

2. (d) The line y = mx meets the given lines in

$P\left(\frac{1}{m+1},\frac{m}{m+1}\right)$ and $Q\left(\frac{3}{m+1},\frac{3m}{m+1}\right)$

Hence equation of L_1 is

$y - \frac{m}{m+1} = 2\left(x - \frac{1}{m+1}\right)$

$\Rightarrow y - 2x - 1 = -\frac{3}{m+1}$ ... (1)

and that of L_2 is

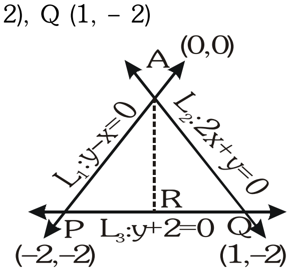
$y - \frac{3m}{m+1} = -3\left(x - \frac{3}{m+1}\right)$

$\Rightarrow y + 3x - 3 = \frac{6}{m+1}$ ... (2)

From (1) and (2) $\frac{y-2x-1}{y+3x-3} = -\frac{1}{2}$

$\Rightarrow x - 3y + 5 = 0$ which is a straight line.

5. (b) Point of intersection of L_1 and L_2 is A(0, 0). Also P(– 2, – 2), Q (1, – 2)



$\because$ AR is the bisector of ∠PAQ, therefore R divides PQ in the same ratio as AP : AQ.

Thus PR : RQ = AP : AQ

$= 2\sqrt{2} : \sqrt{5}$

$\therefore$ Statement-1 is true.

Statement-2 is clearly false.

6. x + 3y – 5 = 0, 3x–xy –1=0,5x+2y –12 = 0

(A) For concurrency

$$\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix} = 0$$

$\Rightarrow$ (12k + 2) – 3(–36 + 5) – 5(6 + 5k) = 0

$\therefore$ k = 5

(B) For parallel

either $-\frac{1}{3} = \frac{3}{k}$ $\therefore$ k = –9

or $-\frac{5}{2} = \frac{3}{k}$ $\therefore$ k = $\frac{-6}{5}$

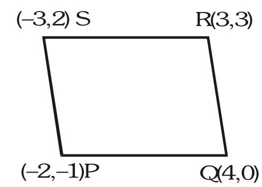
(C) They will form triangle

when k ≠ 5, –9, $\frac{-6}{5}$

(D) They will not form triangle

when k = 5, –9, $\frac{-6}{5}$

7. $\overrightarrow{PQ} = 6\tilde{i} + \tilde{j}$



$\overrightarrow{SR} = 6\tilde{i} + \tilde{j}$

$\therefore$ $\overrightarrow{PQ} = \overrightarrow{SR}$

$\overrightarrow{PS} = -\tilde{i} + 3\tilde{j}$

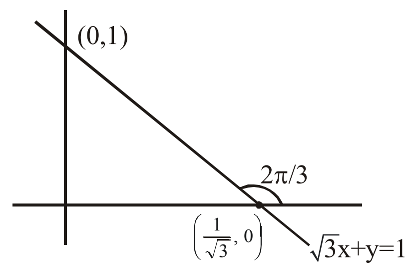
$\overrightarrow{QR} = -\tilde{i} + 3\tilde{j}$

$\therefore$ $\overrightarrow{PS} = \overrightarrow{QR}$

But $\overrightarrow{PQ}.\overrightarrow{PS}$ = – 6 + 3 = –3 ≠ 0 & $|\overrightarrow{PQ}| \neq |\overrightarrow{PS}|$

$\Rightarrow$ PQRS is a parallelogram but neither a rhombus nor a rectangle.

8.



Line L has two possible slopes with inclination;

$\theta = \frac{\pi}{3}$, $\theta = 0$

$\therefore$ equation of line L when $\theta = \frac{\pi}{3}$,

$y + 2 = \sqrt{3}(x-3)$

$\Rightarrow$ $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

equation of line L when θ = 0, y = –2 (rejected)

$\therefore$ required line L is $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

9. Point of intersection of both lines is

$\left(-\frac{c}{(a+b)}, -\frac{c}{(a+b)}\right)$

Distance between $\left(-\frac{c}{(a+b)}, -\frac{c}{(a+b)}\right)$ & (1,1) is

Distance $= \sqrt{\frac{(a+b+c)^2}{(a+b)^2} \times 2} < 2\sqrt{2}$

a + b + c < 2(a + b)

a + b – c > 0

According to given condition option (C) also correct.