

UNIT # 12

PERMUTATION & COMBINATION AND PROBABILITY

PERMUTATION & COMBINATION

EXERCISE - 01

CHECK YOUR GRASP

1. Total possible words – words do not begin or terminate with vowel

$$\text{Total words} = 5! = 120$$

$$\text{Words which do not begin and terminate with vowel} = 3 \times 3 \times 2 \times 1 \times 2 = 36$$

$$\text{Desired words} : 180 - 36 = 84$$

II-Method → words which begin with vowel

$$(A/I) = 4! \times 2 = 48 \text{ ways} \rightarrow \text{say} = n(A)$$

Similarly words terminating with vowel

$$= 4! \times 2 = 48 \text{ ways} \rightarrow \text{say} = n(B)$$

Now exclude words which begin as well as terminates with vowel

$$2 \times 3 \times 2 \times 1 \times 1 = 12 \text{ ways} \rightarrow n(A \cap B)$$

Desired number of words :-

$$48 + 48 - 12 = 84 \text{ ways}$$

$$(\because n(A \cup B) = n(A) + n(B) - n(A \cap B))$$

3. Number of selections of 4 consonants out of 7 is 7C_4

Number of selections of 2 vowels from 4 is 4C_2

Arrangement of words in $6!$ ways

$$\text{Desired words} : {}^7C_4 \times {}^4C_2 \times 6! = 151200$$

7. Make cases when all 5 boxes are filled by:

Case 1 : identical 5 red balls

$${}^5C_5 \rightarrow 1 \text{ way}$$

Case 2 : 4 identical red balls and 1 blue ball

$${}^5C_1 = 5 \text{ ways}$$

Case 3 : 3 blue and 2 red balls i.e. xRxRx

$$\Rightarrow 4 \text{ gaps, for 2 blue balls}$$

$$\therefore {}^4C_2 = 6 \text{ ways}$$

Case 4 : 2 red and 3 blue balls i.e. xRxRx \Rightarrow 3 gaps, 3 blue balls

$$\Rightarrow {}^3C_3 = 1 \text{ way}$$

\therefore Total number of ways are $1+5+6+1 = 13$ ways

9. EEQUU

$$\text{Words starting with E} \rightarrow \frac{4!}{2!}$$

$$\text{Words starting with QE} \rightarrow \frac{3!}{2!}$$

next word will be QUEEU $\rightarrow 1$

and finally QUEUE $\rightarrow 1$

$$\text{Rank is } 12 + 3 + 1 + 1 = 17^{\text{th}}$$

12. For a number to be divisible by 5, 5 or 0 should be at units place.

\therefore Unit place can be filled by 2 ways

Remaining digits can be filled in $\frac{6!}{3! \times 2!}$ ways.

$$\therefore \text{Total ways} = \frac{2 \times 6!}{3! \times 2!}$$

But these arrangements also include cases where 0 is at millions place and 5 at units place, which are undesirable cases

$$\Rightarrow \frac{5!}{3! \times 2!} \text{ ways (undesirable)}$$

subtract it from total ways.

$$\therefore \text{Desired ways} = 2 \times \frac{6!}{3! \times 2!} - \frac{5!}{3! \times 2!} = 110$$

EXERCISE - 02

BRAIN TEASERS

1. $I_H \rightarrow$ Indian husband, $I_W \rightarrow$ Indian wife

$A_H \rightarrow$ American husband, $A_W \rightarrow$ American wife

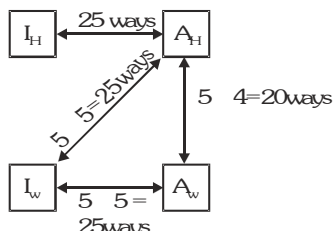
Case 1 : Hand shaking occurring between same nationals and same genders (M/F)

$$I_H - I_H \rightarrow {}^5C_2 = 10 \text{ ways}$$

Similarly for $I_W - I_W$, $A_W - A_W$, $A_H - A_H$

$$\text{Total ways } 10 \times 4 = 40.$$

Case 2 : All other possible hand shakes



Hence total number of handshakes

$$= (25 \times 3 + 40) + (20) = 135$$

Method-II

Total number of handshakes possible ${}^{20}C_2$

undesirable handshakes : ${}^{10}C_2 + {}^{10}C_1$

$$\text{Hence, Desired ways} = {}^{20}C_2 - ({}^{10}C_2 + {}^{10}C_1)$$

3. **Case 1** : When all n red balls are taken but no green ball.

Only 1 arrangement is possible.

Case 2 : n red balls and 1 green balls

$$\text{Number of arrangement} = \frac{n+1}{n \times 1}$$

Case 3 : n red balls and 2 green balls

$$\text{Number of arrangement} = \frac{n+2}{n \times 2}$$

case $m + 1$: n red balls and m green balls

$$\text{Number of arrangements} = \frac{n+m}{n \times m}$$

add all cases

$$1 + \frac{n+1}{n} + \frac{n+2}{n} + \dots + \frac{n+m}{n}$$

$${}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \dots + {}^{n+m}C_m$$

$${}^{n+2}C_1 + {}^{n+2}C_2 + \dots + {}^{n+m}C_m$$

$$(\therefore {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$$

$${}^{n+3}C_2 + {}^{n+3}C_3 + \dots + {}^{n+m}C_m$$

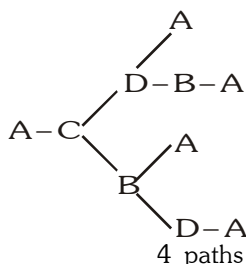
Finally we get the sum as : ${}^{m+n+1}C_m$

5. Maximum number of matches possible, if India win the series are 9

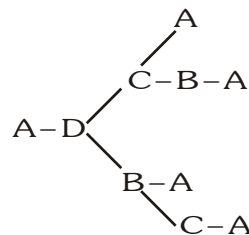
Out of these 9 matches we are required to choose 5 matches that India may win :

$$\Rightarrow \text{Total ways} : {}^9C_5$$

6. Paths are shown as :-



Similarly if we start from A towards B we get another 4 paths.



Similarly if we start from A towards B

Again 4 paths

$$\therefore \text{Total different paths} = 4 \times 3 = 12$$

$$\text{II Method} \rightarrow {}^3C_1 \cdot {}^2C_1 \cdot {}^2C_1 = 12$$

7. Case 1 : Mr. B & Miss C are in committee and Mr. A is excluded

$$\Rightarrow {}^4C_2 \cdot {}^4C_1 = 24 \text{ ways}$$

(Men) (Women)

$$\text{Case 2 : Mr. B not there} : {}^5C_3 \cdot {}^5C_2 = 100 \text{ ways}$$

(Men) (Women)

$$\text{Total ways} = 24 + 100 = 124$$

EXERCISE - 03

Match the column :

1. (A) Each box (say B_1, B_2, B_3) will have at least one ball.

Now the ways for placing other 2 identical balls in 3 different boxes are :-

$$\frac{(2+3-1)!}{2!(3-1)!} = 6 \quad \left(\therefore \frac{(n+r-1)!}{n!(r-1)!} \right)$$

- (B) **Case 1** : 5 balls can be divided in 3 groups having 2 balls each in 2 boxes and 1 ball for in third box (2, 2, 1)

$$\text{ways} : \frac{5!}{(1!)(2!)^2 \times 2!} = 15$$

Case 2 : Division can also be 3 in one box and 1 each in remaining 2 boxes (3, 1, 1)

$$\text{ways} : \frac{5!}{3! \times (1!)^2 \times 2!} = 10$$

Hence total ways = $10 + 15 = 25$

- (C) Only 2 arrangements are possible.

(1) 2 balls each in 2 boxes & remaining ball in other box (2, 2, 1)

(2) 3 balls in 1 box and 1 ball each in other boxes (3, 1, 1)

- (D) Same cases as that of in part B with arrangements.

MISCELLANEOUS TYPE QUESTIONS

Comprehension # 02

1. Exponent of 7 in $100!$ -

$$\left[\frac{100}{7} \right] + \left[\frac{14}{7} \right] = 14 + 2 = 16$$

exponent of 7 in $50!$

$$\left[\frac{50}{7} \right] + \left[\frac{7}{7} \right] = 8$$

$$\text{Exponent of 7 in } {}^{100}C_{50} = \frac{100!}{50!50!} = \frac{7^{16}}{7^8 7^8} = 7^0$$

\therefore exponent of 7 will be 0.

2. Product of 5's & 2's constitute 0's at the end of a number \Rightarrow No. of 0's in $108!$

= exponent of 5 in $108!$

(Note that exponent of 2 will be more than exponent of 5 in $108!$)

$$\Rightarrow \left[\frac{108}{5} \right] + \left[\frac{21}{5} \right] = 21 + 4 = 25$$

3. As $12 = 2^2 \cdot 3$, here we have to calculate exponent of 2 and exponent of 3 in $100!$

exponent of 2

$$= \left[\frac{100}{2} \right] + \left[\frac{50}{2} \right] + \left[\frac{25}{2} \right] + \left[\frac{12}{2} \right] + \left[\frac{6}{2} \right] + \left[\frac{3}{2} \right] = 97$$

$$\text{exponent of 3} = \left[\frac{100}{3} \right] + \left[\frac{33}{3} \right] + \left[\frac{11}{3} \right] + \left[\frac{3}{3} \right] = 48$$

Now, $12 = 2^2 \cdot 3$

we require two 2's & one 3

\therefore exponent of 3 will give us the exponent of 12 in $100!$ i.e. 48

EXERCISE - 04[A]**CONCEPTUAL SUBJECTIVE EXERCISE**

2. Selecting 3 horses out of ABC A'B'C' is 6C_3 ways
 When AA' is always selected among (ABC A'B'C')
 Remaining (BB'CC') can be selected in 4C_1 ways
 similarly, when BB' and CC' is selected
 \therefore Undesirable ways will be $({}^4C_1) \cdot 3$
 using, total ways – undesirable ways = desired ways
 we get

$({}^6C_3 - ({}^4C_1) \cdot 3) \rightarrow$ This is selection of 3 horses among (ABC A'B'C') under given condition.

Remaining 3 can be selected in ${}^{10}C_3$ ways.

Hence, desired ways will be $[{}^6C_3 - {}^4C_1 \cdot 3] {}^{10}C_3 = 792$

Method II : Select one horse each from AA', BB' and CC' hence ${}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1$ ways. Now select 3 horses from remaining 10 horses in ${}^{10}C_3$ ways.
 Total ways = ${}^{10}C_3 \cdot {}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1$

3. **Case 1 :** When all digits are same : 9C_1 (excluding 0)

Case 2 : When digits are different and 0 excluded.

(a) Selecting 2 numbers in 9C_2 ways

(b) Each digit can be filled in 2 ways hence

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4 \text{ way}$$

(c) Undesirable case : when a particular digit is same ($2 \cdot 1 = 2$ ways) (case 1)

$$\therefore {}^9C_2 (2^4 - 2) \text{ ways}$$

Case 3 : When digits are different and 0 is included

(a) other digit can be chosen in 9C_1 ways

(b) 0 can't be placed at ten thousand's place, hence selected digit should be fixed at this place remaining 3 digits can be filled with $2 \cdot 2 \cdot 2 = 2^3$ ways.

(c) Undesirable case : When all the 4 digits gets filled with selected digit only (0 not included) = 1 way

$$\text{hence no. of ways will be : } {}^9C_1 (2^3 - 1)$$

$$\therefore \text{Total desired ways :- } {}^9C_1 + {}^9C_2 (2^4 - 2) + {}^9C_1 (2^3 - 1) = 576 \text{ ways}$$

7. Husband – H, Wife – W

Given :

Relatives of husband (H) (a) Ladies (L_H) = 4

(b) Gentlemen (G_H) = 3

Relatives of Wife (W) (a) Ladies (L_W) = 3

(b) Gentlemen (G_W) = 4

Case 1 : Selecting ($3L_H$) and ($3G_W$)

$$\text{ways : } {}^4C_3 \cdot {}^4C_3 = 16$$

Case 2 : Selecting ($3G_H$) and ($3L_W$)

$$\text{ways : } {}^3C_3 \cdot {}^3C_3 = 1$$

Case 3 : Selecting ($2L_H$ & $1G_H$) & ($1L_W$ & $2G_W$)

$$\text{ways : } {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 = 324$$

Case 4 : Selecting ($1L_H$ & $2G_H$) & ($2L_W$ & $1G_W$)

$$\text{ways : } {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 = 144$$

Add all cases we get : 485 ways

8. Step 1st : Arrange 5 boys in 5! ways

Step 2nd : Select 2 gaps from 6 gaps for 4 girls (2girls for each gap) in 6C_2 ways.

Step 3rd : Select 2 girls to sit in one of the gaps and other 2 in remaining selected gaps = 4C_2 ways

Step 4 : Arrange 1st, 2 girls in 2! and other 2 in 2! ways

$$\text{Hence, total ways} \rightarrow 5! \cdot {}^6C_2 \cdot {}^4C_2 \cdot 2 \cdot 2 = 43200$$

13. Distribute 15 candies among.

Ram (R) + Shyam(S) + Ghanshyam(G) + Balram(B)

with condition given : $R+S+G+B=15$ & $R \leq 5$ & $S \geq 2$

After giving 2 to Shyam, remaining candies $15-2=13$

Now distribute 13 candies in

$$R, S, G, B \text{ in } \frac{13+4-1}{13 \cdot 3} = {}^{16}C_3 \text{ ways}$$

In ${}^{16}C_3$ ways, we have to remove undesirable ways, when $R > 5$

$$\text{Undesirable ways : } R > 5 \Rightarrow R \geq 6$$

give at least 6 to R and 2 to S and distribute remaining between R, S, G, B

$$15 - (2 + 6) = 7 \text{ remaining can be distributed}$$

$$\text{between R, S, G, B in } = \frac{7+4-1}{7 \cdot 4-1} = {}^{10}C_3 \text{ ways}$$

${}^{10}C_3$ are the undesirable cases

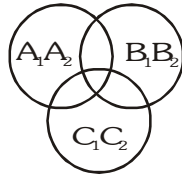
$$\text{Desired ways} = {}^{16}C_3 - {}^{10}C_3 = 440$$

18. Total number of ways = Put exactly one ball in its box and then dearrange remaining balls.

$$= {}^5C_1 \cdot 4! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right] = 5 \cdot 9 = 45$$

EXERCISE - 04[B]**BRAIN STORMING SUBJECTIVE EXERCISE**

3. (i) Total ways = 10!
 undesirable cases : when 2 Americans are together (A_1A_2)
 or two British are together (B_1B_2) or two Chinese are together (C_1C_2)
 we plot them on Venn diagram :



we use,

$$\begin{aligned} n(A_1A_2 \cup B_1B_2 \cup C_1C_2) &= n(A_1A_2) + n(B_1B_2) \\ &+ n(C_1C_2) - n[(A_1A_2) \cup (B_1B_2)] \\ &- n[(B_1B_2) \cup (C_1C_2)] - n[(C_1C_2) \cup (A_1A_2)] \\ &+ n[(A_1A_2) \cap (B_1B_2) \cap (C_1C_2)] \end{aligned}$$

where $n(A_1A_2)$ denotes \rightarrow when 2 Americans are together = $9! \cdot 2!$

correspondingly for B_1B_2 & C_1C_2

$n[(A_1A_2) \cup (B_1B_2)]$ denotes when 2 Americans and 2 Britishmen are together = $8! \cdot 2! \cdot 2!$

correspondingly same for others.

$n[(A_1A_2) \cap (B_1B_2) \cap (C_1C_2)]$ denotes when 2 Americans, 2 Britishmen and 2 Chinese are together = $7! \cdot 2! \cdot 2! \cdot 2! = 86$

Put values we get

$$\begin{aligned} n(A_1A_2 \cup B_1B_2 \cup C_1C_2) &= 9! \cdot 2! \cdot 3 - 8! \cdot 2 \cdot 2 \cdot 3 + 8! \\ &= 8!(43) \end{aligned}$$

These are undesired ways

$$\text{Desired ways} = 10! - 8!(43) = 8!(47)$$

- (ii) Now they are on a round table

$$\text{Total ways} = (n-1)! = (10-1)! = 9!$$

Undesired ways :

$$\begin{aligned} n(A_1A_2 \cup B_1B_2 \cup C_1C_2) &= 8! \cdot 2! \cdot 3 - 7! \cdot 2! \cdot 2! \cdot 3 + 6! \cdot 2! \cdot 2! \cdot 2! \\ &= 6! \cdot 4 [7 \cdot 2 \cdot 2 \cdot 3 - 7 \cdot 3 + 2] \\ &= 6! \cdot 260 \end{aligned}$$

$$\text{Desired ways} = 9! - 6! \cdot 260$$

$$= (244) \cdot 6! \text{ ways}$$

6. (a) Selection of r things out of $n+1$ different things = Selection of r things out of $n+1$ different things, when a particular thing is excluded + a particular thing is included.
 (b) Selection of r things out of not $m+n$ different things can be made by selecting x thing from m and y thing from such that $x+y=r$
 & $(x, y) = (0, r), (1, r-1), (2, r-2), \dots, (r, 0)$

9. 2 clerks who prefer Bombay are to be sent to 2 different companies in Bombay,

and Out of remaining 5 clerks (excluding 3 clerks who prefer for outside) 2 clerks are chosen in 5C_2 ways.

Now these 4 can be sent to 2 different companies into 2 groups of 2 each in 4C_2 ways

$$\Rightarrow {}^5C_2 \cdot {}^4C_2$$

Now for outside companies we have 6 clerks remaining we select them as (2 for each company)

$${}^6C_2 \cdot {}^4C_2 \cdot {}^2C_2$$

Desired ways = $({}^5C_2 \cdot {}^4C_2) ({}^6C_2 \cdot {}^4C_2 \cdot {}^2C_2) = 5400$ ways.

11. Total cases of selecting 8 men out of 11 is ${}^{11}C_8$

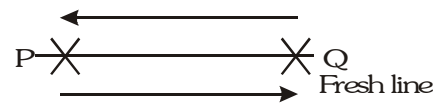
Undesirable case : When all the 5 men, who can only row on stroke side are selected then other 3 men can be selected in 6C_3 ways.

$$\text{Hence, } {}^{11}C_8 - {}^6C_3 = 145 \text{ Ans.}$$

12. Step 1st : Select 2 lines out of n lines in nC_2 ways to get a point (say p).

Step-2nd : Now select another 2 lines in ${}^{n-2}C_2$ ways, to get another point (say Q)

Step-3rd : When P and Q are joined we get a fresh line.



But when we select P first then Q and Q first then P we get same line.

$$\therefore \frac{{}^nC_2 \times {}^{n-2}C_2}{2} \text{ Fresh lines}$$

EXERCISE - 05 [A]**JEE-[MAIN] : PREVIOUS YEAR QUESTIONS**

1. Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the first place with 0 in each of the remaining places.
After fixing 1st place, the second place can be filled by any of the 5 numbers. Similarly third place can be filled up in 5 ways and 4th place can be filled up in 5 ways. Thus there will be $5 \times 5 \times 5 = 125$ ways in which 1 will be in first place but this include 1000 also hence there will be 124 numbers having 1 in the first place. Similarly 125 for each 2 or 3. One number will be in which 4 in the first place and i.e. 4000. Hence the required numbers are $124 + 125 + 125 + 1 = 375$ ways.
2. We know that a five digit number is divisible by 3, if and only if sum of its digits (= 15) is divisible by 3. Therefore we should not use 0 or 3 while forming the five digit numbers. Now, (i) in case we do not use 0 the five digit number can be formed (from the digit 1, 2, 3, 4, 5) in 5P_5 ways.
(ii) In case we do not use 3, the five digit number can be formed (from the digit 0, 1, 2, 4, 5) in ${}^5P_5 - {}^4P_4 = 5! - 4! = 120 - 24 = 96$ ways.
 \therefore The total number of such 5 digit number $= {}^5P_5 + ({}^5P_5 - {}^4P_4) = 120 + 96 = 216$
4. No. of ways in which 6 men can be arranged at a round table = $(6 - 1)!$
Now women can be arranged in $6!$ ways.
Total Number of ways = $6! \times 5!$
5. As for given question two cases are possible
(i) Selecting 4 out of first 5 question and 6 out of remaining 8 questions = ${}^5C_4 \times {}^8C_6 = 140$ choices
(ii) Selecting 5 out of first 5 questions and 5 out of remaining 8 questions = ${}^5C_5 \times {}^8C_5 = 56$ choices.
 \therefore Total no. of choices = $140 + 56 = 196$
7. Number of ways to arrange in which vowels are in alphabetical order = $\frac{6!}{2!} = 360$
8. Number of ways = ${}^{n-1}C_{r-1} = {}^{8-1}C_{3-1} = {}^7C_2 = 21$
11. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$
12. Number of ways = $\frac{12!}{(4!)^3 \cdot 3!} \cdot 3! = \frac{12!}{(4!)^3}$
14. The no. of ways to select 4 novels & 1 dictionary from 6 different novels & 3 different dictionary are ${}^6C_4 \times {}^3C_1$
and to arrange these things in shelf so that dictionary is always in middle _ _ D _ _ are 4!
Required No. of ways ${}^6C_4 \times {}^3C_1 \times 4! = 1080$
15. Urn A \rightarrow 3 Red balls
Urn B \rightarrow 9 Blue balls
So the number of ways = selection of 2 balls from urn A & B each.
 $= {}^3C_2 \times {}^9C_2 = 108$
16. $B_1 + B_2 + B_3 + B_4 = 10$
St - 1 : $B_1 \geq 1, B_2 \geq 1, B_3 \geq 1, B_4 \geq 1$
so no. of negative integers solution of equation $x_1 + x_2 + x_3 + x_4 = 10 - 4 = 6$
 ${}^{6+4-1}C_{4-1} = {}^9C_3$
St - 2 : selection of 3 places from out of 9 places = 9C_3
Both statements are true and correct explanation
17. $N = {}^{10}C_3 - {}^6C_3 = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} - \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$
 $= 120 - 20 = 100$
 $N \leq 100$
18. W^{10}, G^9, B^7
selection of one or more balls
 $= (10 + 1)(9 + 1)(7 + 1) - 1$
 $= 11 \times 10 \times 8 - 1 = 879$
19. (A, B)
 $\uparrow \quad \uparrow$
 $2 \quad 4 = 8$
 ${}^8C_3 + {}^8C_4 + \dots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
 $= 256 - 37 = 219$
20. $T_n = {}^nC_3 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 10$
 $(n + 1)n(n - 1) - n(n - 1)(n - 2) = 60$
 $n(n - 1) = 20$
 $n = 5$

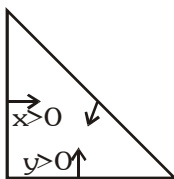
4. $x + y < 21$

$x + y \leq 20$

$x + y \leq 18$ ($\because x > 0$ & $y > 0$)

Introducing new variable t

$x + y + t = 18$



Now dividing 18 identical things among 3 persons.

$$= {}^{18+3-1}C_{3-1} = \frac{18+3-1}{18 \cdot 3-1} = 190$$

5. Total number of ways of distributing n^2 objects into n groups, each containing n objects

$$= \frac{(n^2)!}{(n!)^n n!} = \frac{(n^2)!}{(n!)^n} = \text{integer}$$

(Since number of ways are always integer)

8. Since, r, s, t are prime numbers.

\therefore Selection of p and q are as under

p	q	number of ways
r^0	r^2	1 way
r^1	r^2	1 way
r^2	r^0, r^1, r^2	3 ways
\therefore Total number of ways to select $r = 5$		
s^0	s^4	1 way
s^1	s^4	1 way
s^2	s^4	1 way
s^3	s^4	1 way
s^4	s^0, s^1, s^2, s^3, s^4	5 ways

\therefore Total number of ways to select $s = 9$.

Similarly total number of ways to select $t = 5$
number of ways = $5 \cdot 9 \cdot 5 = 225$.

11. **Ans. (D)**

Case- I : The number of elements in the pairs can be 1,1; 1,2; 1,3; 2,2

$$= {}^4C_2 + {}^4C_1 \cdot {}^3C_2 + {}^4C_1 \cdot {}^3C_3 + \frac{{}^4C_2 \cdot {}^2C_2}{2} = 25$$

Case- II : Number of pairs with ϕ as one of subsets
 $= 2^4 = 16$

\therefore Total pairs = $25 + 16 = 41$

- 12 Balls can be distributed as 1, 1, 3 or 1, 2, 2 to each person.

When 1, 1, 3 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!1!3!} \cdot \frac{1}{2!} \cdot 3! = 60$$

When 1, 2, 2 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!2!2!} \cdot \frac{1}{2!} \cdot 3! = 90$$

$$\therefore \text{total} = 60 + 90 = 150$$

Paragraph for Question 13 and 14 :

For a_n

The first digit should be 1

For b_n

$$\underbrace{1 \text{ --- } \dots \text{ --- } 1}_{(n-2 \text{ Places})}$$

Last digit is 1. so b_n is equal to number of ways of a_{n-1} (i.e. remaining $(n-1)$ places)

$$b_n = a_{n-1}$$

For c_n

Last digit is 0 so second last digit must be 1

$$\text{So } c_n = a_{n-2}$$

$$b_n + c_n = a_n$$

$$\text{So } a_n = a_{n-1} + a_{n-2}$$

$$\text{Similarly } b_n = b_{n-1} + b_{n-2}$$

13. **Ans.(B)**

$$a_1 = 1, a_2 = 2$$

$$\text{So } a_3 = 3, a_4 = 5, a_5 = 8$$

$$\Rightarrow b_6 = a_5 = 8$$

14. **Ans.(A)**

$$a_n = a_{n-1} + a_{n-2}$$

$$\text{put } n = 17$$

$$a_{17} = a_{16} + a_{15} \quad (\text{A is correct})$$

$$c_n = c_{n-1} + c_{n-2}$$

$$\text{So put } n = 17$$

$$c_{17} = c_{16} + c_{15} \quad (\text{B is incorrect})$$

$$b_n = b_{n-1} + b_{n-2}$$

$$\text{put } n = 17$$

$$b_{17} = b_{16} + b_{15} \quad (\text{C is incorrect})$$

$$a_{17} = a_{16} + a_{15}$$

while (D) says $a_{17} = a_{15} + a_{15}$ (D) is incorrect