UNIT # 03
FUNCTION

EXERCISE - 01 CHECK YOUR GRASP

4. $f(x) = \sqrt{\log\frac{(5x-x^2)}{6}}$

$\log\frac{5x-x^2}{6} \geq 0$

$\Rightarrow \quad \frac{5x-x^2}{6} \geq 1 \quad \Rightarrow \quad x^2 - 5x + 6 \leq 0$

$\Rightarrow \quad (x-2)(x-3) \leq 0 \quad \Rightarrow \quad 2 \leq x \leq 3$

So domain $\in [2, 3]$

8. $f(x) = {}^{7-x}P_{x-3}$

For domain

$7 - x \geq 0, \ \& \ x - 3 \geq 0 \quad \& \ 7 - x \geq x - 3$

$x \leq 7, \ \& \ x \geq 3 \quad 2x \leq 10$

$x \leq 5$

$x \in \{3, 4, 5\}$

Range $\in \{f(3), f(4), f(5)\}$

Range $\in \{1, 3, 2\}$

12. $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ ....(i)

replacing x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1$ ....(ii)

Solve (i) and (ii) we get

$f(x^2) = \frac{3-2x^4-2x^2}{5x^2}$

14. $f(x+1) - f(x) = 8x + 3$

$f(0+1) - f(0) = 3$ (put x = 0)

$\Rightarrow \quad (b + c + d) - d = 3$

$\Rightarrow \quad b + c = 3$ ....(i)

Also $f(-1+1) - f(-1) = -8 + 3$ (put x = –1)

$\Rightarrow \quad f(0) - f(-1) = -5 \quad \Rightarrow \quad d - (b - c + d) = -5$

$\Rightarrow \quad -b + c = -5$ ....(ii)

from (i) and (ii)

$b = 4, \quad c = -1$

19. $f(x) = x - [x] + (x + 1) - [x + 1] + \ldots\ldots\ldots$

$(x + 99) - [x + 99]$

$= x - [x] + x - [x] + \ldots\ldots\ldots + x - [x]$

$= 100(x - [x]) = 100\{x\}$

$f(\sqrt{2}) = 100\{\sqrt{2}\} = 41$

23. Hint : $f(x) = \frac{e^{|x|}-e^{-x}}{e^x+e^{-x}} = \begin{cases} \frac{e^x-e^{-x}}{e^x+e^{-x}} & x > 0 \\ 0 & x \leq 0 \end{cases}$

and $\frac{e^x-e^{-x}}{e^x+e^{-x}} > 0 \ \forall \ x > 0$

25. $fog(x) = \log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right)$

$= \log\left(\frac{(1+x)^3}{(1-x)^3}\right) = 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$

28. $f(x) = \sin\sqrt{[a]}\ x$

period of sinx = 2π

$\Rightarrow$ period of $f(x) = \frac{2\pi}{\sqrt{[a]}} = \pi$

$\Rightarrow \quad \sqrt{[a]} = 2 \quad \Rightarrow \quad [a] = 4 \quad \Rightarrow \quad a \in [4, 5)$

33. Put y = – x, we get f(x) = –x also f(0) = 0

f(x + y) = f(x) + f(y) is an odd function so it is symmetric about origin.

36. $f(x+1) + f(x+3) = K \quad \forall \ x$

put x = –1

$f(0) + f(2) = K$ .....(i)

put x = 1 $\quad f(2) + f(4) = K$ .....(ii)

from (i) & (ii)

$f(4) = f(0) = 0 \quad \Rightarrow$ period = 4

39. $f(x) = 2^{x(x-1)}$

It is one-one onto function

$\log_2 y = x(x-1)$

$\Rightarrow \quad x^2 - x - \log_2 y = 0$

$x = \frac{1 \pm \sqrt{1+4\log_2 y}}{2}$

$f^{-1}(x) = \frac{1+\sqrt{1+4\log_2 x}}{2}$

42. (A) $\sin x + \cos x = \sqrt{2}\sin\left(\frac{\pi}{4}+x\right) \rightarrow$ Periodic

(B) $\cos x \rightarrow$ period 2π

$\left\{\frac{x}{\pi}\right\} \rightarrow$ period π

So period of $\cos x + \left\{\frac{x}{\pi}\right\} = 2\pi$

(C) $\cos \pi x \rightarrow$ period 2

$\{2x\} \rightarrow$ period $\frac{1}{2}$

so period of $\cos \pi x + \{2x\} = 2$

(D) $\ell n\{x\} \rightarrow$ period 1

$\sin 2x \rightarrow$ period π

$\ell n\{x\} + \sin 2x \rightarrow$ no period

EXERCISE - 02 BRAIN TEASERS

3. $f(e^x) + f(\ln|x|)$ $x \in (0, 1)$

Now $0 < e^x < 1$ & $0 < \ell n |x| < 1$

$\Rightarrow$ $-\infty < x < 0$ ......(i)

$\Rightarrow$ $1 < |x| < e$

$\Rightarrow$ $(-e,-1) \cup (1,e)$....(ii)

from (i) and (ii)

domain of x is $(-e, -1)$

8. $y = \cos(K \sin x)$

$\frac{\cos^{-1} y}{K} = \sin x \Rightarrow -1 \le \frac{\cos^{-1} y}{K} \le 1$

$\Rightarrow$ $-K \le \cos^{-1} y \le K$

Now $\cos^{-1} y \in [0, \pi]$

$\Rightarrow$ $K = 4$

10. $g(x)\,g(y) = g(x) + g(y) + g(xy) - 2$

put $x = 2$ & $y = 1$

$g(2)\,g(1) = g(2) + g(1) + g(2) - 2$

$\Rightarrow$ $4g(1) = 8 \Rightarrow g(1) = 2$

$g(x)\,g(y) = g(x) + g(y)$, now put $y = \frac{1}{x}$

Now $g(x)\,g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right)$

$g(x) = 1 \pm x^n$

$\therefore$ $5 = 1 \pm 2^n$ ($\because$ $g(2) = 5$)

so, $n = 2$

Now $g(3) = 1 + 3^2 = 10$

12. $\log_{x^2}(x) \ge 0$ & $x > 0,\ x \ne \pm 1$

$\therefore$ $x \in (0, 1) \cup (1, \infty)$

16. put $x = 1$

$2f(1) + 1f(1) - 2f(\,|\sqrt{2}\ \sin\frac{5\pi}{4}|) = -1$

$\Rightarrow$ $3f(1) - 2f(1) = -1 \Rightarrow f(1) = -1$

Now put $x = 2$

$2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4\cos^2\pi + 2\cos\frac{\pi}{2}$

$\Rightarrow$ $2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4$

$\Rightarrow$ $f(2) + f\left(\frac{1}{2}\right) = 1$..........(i)

Now put $x = 1/2$ we get

$4f\left(\frac{1}{2}\right) + f(2) = 1$ ....(ii)

from (i) and (ii)

$f\left(\frac{1}{2}\right) = 0$ & $f(2) = 1$

18. $f(x) = \sin\left[\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right]$

f(x) will be defined if $\frac{\sqrt{4-x^2}}{1-x} > 0$ & $4 - x^2 > 0$

$\Rightarrow$ $-2 < x < 1$ & $-\infty < \log\frac{\sqrt{4-x^2}}{1-x} < \infty$

$-1 \le \sin\left[\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right] < 1$

so range of f (x) is $[-1, 1]$

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Match the Column :

4. (A) If putting $f(x) = 0$ then we get

$x = 1, 2,\ldots\ldots,11$ (many one)

and $f(x)$ is a polynomial function of degree odd defined from R to R which is always onto.

Hence $f(x)$ is many one-onto

(B) $f'(x) = \frac{5}{(3x+4)^2} > 0\ \forall\ x \in D_f$ (one - one)

$y = \frac{2x+1}{3x+4}$

$x = \frac{1-4y}{3y-2} \Rightarrow y \ne \frac{2}{3}$

$\therefore$ Range of f is $R-\left\{\frac{2}{3}\right\} \subset$ co-domain (into)

Hence $f(x)$ is one-one - into

(C) putting $x = 0, \pi, 2\pi$ ....

we get same value of $f(x)$ equal to 2 (many-one)

$f(x) = e^{\sin x} + \frac{1}{e^{\sin x}} \Rightarrow f(x) \ge 2\ \forall\ x \in R$

Range of f is $[2, \infty) \subset$ co-domain (into)

Hence $f(x)$ is many one into

(D) $f(x) = \log[(x+1)^2 + 2]$

at $x = 0$ & -2 we get same value of $f(x)$ equal to log3 (many-one)

$f(x) \geq \log 2 \;\; \forall\, x \in R$

Range of f is $[\log 2, \infty) \subset$ co-domain (into)

Hence $f(x)$ is many one-into.

Assertion & Reason :

1. St. I : $f(x) = \begin{cases} x & x \in Q \\ 1-x & x \notin Q \end{cases}$

$\Rightarrow f(f(x)) = f(x)\,,\; f(x) \in Q$

$= 1 - f(x),\; f(x) \notin Q$

$= 1 - (1 - x) = x$

St. II : $f(-x) = -x \qquad x \in Q$

$= 1 + x \qquad x \notin Q$

$\Rightarrow \quad f(-x) \neq \pm f(x)$

Hence neither even not odd

5. Put $\quad x = y = 0 \;\Rightarrow f(0) = 0, 1$

if $\quad f(0) = 0 \quad$ then

putting $y = 0 \;\Rightarrow\; f(x)f(0) - f(0) = 0 + x \;\forall\, x \in R$

$\Rightarrow x = 0 \;\; \forall\, x \in R$ hence contradiction.

if $\quad f(0) = 1$ then by $\;y = 0,\;\; f(x) - 1 = 0 + x.$

$f(x) = x + 1$ which is an injective function having range R so bijective

But every linear function is not bijective as $y = c$.

6. Statement-I :

Let $\;f(x) = \dfrac{1}{x} \quad\Rightarrow\quad f^{-1}(x) = \dfrac{1}{x}$

$f(x) = f^{-1}(x) \Rightarrow \; x \in R_0 \;\; \not\Rightarrow \; f(x) = x$

as $f(x) = x$ holds only on $x = \pm 1$

$\Rightarrow \quad$ statement-I is false

Statement-II :

$f^{-1}(x) = x \;\;\Rightarrow\;\; f(f(x)) = f(x) \;\;\Rightarrow x = f(x)$

Comprehension # 3 :

$f(2 - x) = f(2 + x)$

& $\quad f(20 - x) = f(x)$

$\Rightarrow \quad f(2 - (2 - x)) = f(4 - x)$

& $\quad f(20 - (x + 16)) = f(x + 16)$

$\Rightarrow \quad f(x) = f(4 - x)$

& $\quad f(4 - x) = f(x + 16)$

$\Rightarrow \quad f(x) = f(x + 16)$

1. $f(0) = f(4) = f(16)$

no. of values of $x = 22$

2. If graph is symmetric about $x = a$ then

$f(a + x) = f(a - x)$

$f(16) = f(20) \quad \Rightarrow$ symmetric about $x = 18$

$f(4) = f(32)$

3. $f(0) = f(1) = f(2) = f(3) = f(4) = f(5) = f(6)$

hence period can't be one.

EXERCISE - 04 (A) CONCEPTUAL SUBJECTIVE EXERCISE

3. (a) $f(f(x))\,[1 + f(x)] = -f(x)$

$f(f(x)) = \dfrac{-f(x)}{1+f(x)} \quad\Rightarrow\quad f(x) = \dfrac{-x}{1+x}$

$f(3) = \dfrac{-3}{1+3} = \dfrac{-3}{4}$

(b) $f(x + f(x)) = 4\,f(x) \qquad \&\; f(1) = 4$

$f(1 + f(1)) = 4f(1) \Rightarrow f(1 + 4) = 16$

$f(5) = 16$

Now $\;f(5 + f(5)) = 4\,f(5)$

$f(5 + 16) = 64 = f(21)$

(c) $[f(xy)]^2 = x\,[f(y)]^2 \quad \&\; f(2) = 6$

put $x = 2$ & $y = 1$

$\Rightarrow \quad [f(2 \quad 1)]^2 = 2\,(f(1))^2 \quad\Rightarrow\quad (f(1))^2 = 18$

Now $\;[f(50 \quad 1)]^2 = 50\,[f(1)]^2 = 50 \quad 18$

$f(50) = 30$

(d) $f(x + y) = x + f(y) \quad \& \quad f(0) = 2$

$f(100 + 0) = 100 + f(0) = 102$

4. $f(3) = 1$

$f(3x) = x + f(3x - 3)$

put $x = 1$

$f(3) = 1 + f(0)$

$f(0) = 0$

$f(6) = 2 + f(3) = 3$

$f(9) = 3 + f(6) = 3 + 3 = 6$

$f(12) = 4 + 6 = 10$

hence $f(300) = 1 + 3 + 6 + 1 + \ldots.. \; 100^{th}$ term

$S = 1 + 3 + 6 + 10 \;\ldots\ldots.. + T_n$

$S = 1 + 3 + 6 + \ldots.. + T_n$

$T_n = 1 + 2 + 3 + 4 \;\ldots\ldots\ldots.$ up 100 term

$= \dfrac{100}{2} \quad 101 = 5050$

5. $f(x) = \dfrac{9^x}{9^x + 3}$

$f(1 - x) = \dfrac{9^{1-x}}{9^{1-x} + 3} = \dfrac{3}{3 + 9^x}$

$f(x) + f(1 - x) = 1$

$f\left(\dfrac{1}{2008}\right) + f\left(\dfrac{2007}{2008}\right) = 1 \qquad \ldots\ldots\ldots..(i)$

$f\left(\dfrac{2}{2008}\right) + f\left(\dfrac{2006}{2008}\right) = 1 \qquad \ldots\ldots\ldots(ii)$

....

....

$f\left(\frac{1003}{2008}\right)+f\left(\frac{1005}{2008}\right)=1$ ....(iii)

& $f\left(\frac{1004}{2008}\right)+f\left(\frac{1004}{2008}\right)=1$

$\Rightarrow f\left(\frac{1004}{2008}\right) = \frac{1}{2}$ ....(iv)

add all we get

$f\left(\frac{1}{2008}\right)+f\left(\frac{2}{2008}\right)+f\left(\frac{3}{2008}\right)+\ldots+f\left(\frac{2007}{2008}\right)$

= 1003.5

6. (f) For $\ell n\{x\}$ to be defined $x \neq I$

$\{\ell n\{x\}\} \to (0, 1) \Rightarrow \left[\{\ell n\{x\}\}\right] = 0$

& $2x^2 - 7x + 5 \le 0$

$\Rightarrow (2x-5)(x-1) \le 0 \Rightarrow 1 \le x \le \frac{5}{2}$

& $\frac{1}{\ell n\left(\frac{7}{2}-x\right)}$

$\frac{7}{2} - x > 0 \Rightarrow x < \frac{7}{2}$

also $\frac{7}{2} - x \neq 1 \Rightarrow x \neq \frac{5}{2}$

$\therefore x \in (1, 2) \cup (2, \frac{5}{2})$

(g) $\frac{f}{g}(x) = \frac{\sqrt{x^2-5x+4}}{x+3}$

$\Rightarrow x^2 - 5x + 4 \ge 0$

$(x-4)(x-1) \ge 0$

also $x \neq -3$

so $x \in (-\infty, -3) \cup (-3, 1] \cup [4, \infty)$

(h) $\frac{1}{[x]} \Rightarrow x \notin [0, 1)$

and $\log_{1-\{x\}}(x^2 - 3x + 10)$

$x^2 - 3x + 10 > 0 \Rightarrow x \in R$

$1 - \{x\} > 0 \Rightarrow x \in R$

$1 - \{x\} \neq 1 \Rightarrow x \notin I$

and $2 - |x| > 0 \Rightarrow |x| - 2 < 0$

$\Rightarrow x \in (-2, 2)$

and $\sec(\sin x) > 0 \Rightarrow -1 \le \sin x \le 1$

$\Rightarrow x \in R$

$x \in (-2, -1) \cup (-1, 0) \cup (1, 2)$

8. (a) $y = \log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + 3)$

$\Rightarrow 2\sin(x - \frac{\pi}{4}) + 3 > 0$

$\therefore$ Domain $x \in R$

$\Rightarrow -2 + 3 \le 2\sin(x - \frac{\pi}{4}) + 3 \le 2 + 3$

Range = $\left[\log_{\sqrt{5}} 1, \log_{\sqrt{5}} 5\right]$ = [0, 2]

18. (a) $10^x + 10^y = 10$

$10^y = 10 - 10^x$

$y = \log_{10}(10 - 10^x)$

Domain : $10 - 10^x > 0 \Rightarrow 10 > 10^x$

$\Rightarrow x < 1 \Rightarrow x \in (-\infty, 1)$

20. $f(x) = (a - x^n)^{1/n}$

$f(f(x)) = (a - (f(x))^n)^{1/n}$

$= [a - \{(a - x^n)^{1/n}\}^n]^{1/n} = (a - a + x^n)^{1/n} = x$

so $fof(x) = x$

$\Rightarrow f^{-1}(x) = f(x) = (a - x^n)^{1/n}$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. (a) $f(x) = \sqrt{\frac{1-5^x}{7^{-x}-7}}$

(i) $1 - 5^x \ge 0 \Rightarrow 1 \ge 5^x \Rightarrow x \le 0$

& $7^{-x} - 7 > 0$

$\Rightarrow x < -1$

$\Rightarrow x \in (-\infty, -1)$

(ii) $1 - 5^x \le 0 \Rightarrow x \ge 0$

$7^{-x} - 7 < 0 \Rightarrow x > -1$

$\Rightarrow x \in [0, \infty)$

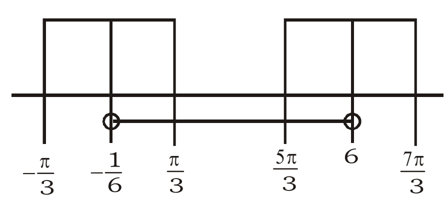
$x \in (-\infty, -1) \cup [0, \infty)$

(c) $f(x) = \frac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6+35x-6x^2}}$

$\Rightarrow \cos x - \frac{1}{2} \ge 0$

$\Rightarrow x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, \frac{7\pi}{3}\right]$

$\Rightarrow 6x^2 - 35x - 6 < 0 \Rightarrow -\frac{1}{6} < x < 6$



$\Rightarrow x \in \left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$.

2. $f(x) = \sin x + \cos(px)$

$f(x + T) = \sin(x + T) + \cos(px + pT)$

$= \sin x + \cos(px)$

$\sin T + \cos pT = f(0) = 1$

$\sin(-T) + \cos pT = 1$

$\Rightarrow \quad 2 \sin T = 0 \quad \Rightarrow \quad T = n\pi$

Now $\sin n\pi + \cos pn\pi = 1$

$\cos pn\pi = 1$

$pn\pi = 2m\pi$

$p = \dfrac{2m}{n}$ i.e. Rational.

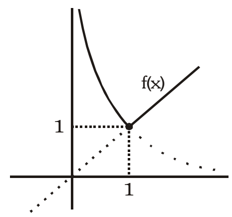
6. $x = \dfrac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = 1$

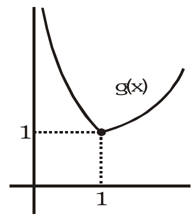
$f(x) = \dfrac{1}{x} \quad ; \quad 0 < x \le 1$

$= x \quad ; \quad x > 1$

$g(x) = f(x)\, f\left(\dfrac{1}{x}\right)$

$\Rightarrow \quad g(x) = \begin{cases} \dfrac{1}{x}.\dfrac{1}{x} & 0 < x \le 1 \\ x.x & x > 1 \end{cases}$





8. $p(x) = (x - 1)\, Q_1(x) + 1$

$p(x) = (x - 4)\, Q_2(x) + 10$

$\Rightarrow (x - 4)\, p(x) = (x - 4)(x - 1)\, Q_1(x) + (x - 4)$

& $(x - 1)\, p(x) = (x - 1)(x - 4)\, Q_2(x) + 10x - 10$

$\Rightarrow \quad p(x) = \dfrac{(x-1)(x-4)}{3}\, [Q_2(x) - Q_1(x)] + \dfrac{9x-6}{3}$

$\Rightarrow \quad r(x) = \dfrac{9x-6}{3} = 3x - 2 = 3 \quad 2006 - 2 = 6016.$

10. $f(x) = (x + 1)(x + 2)(x + 3)(x + 4) + 5$

$x \in [-6, 6]$

$= (x^2 + 5x + 4)(x^2 + 5x + 6) + 5$

Let $t = x^2 + 5x$

$(t + 4)(t + 6) + 5 = t^2 + 10t + 24 + 5$

$= t^2 + 10t + 29$

$= (t + 5)^2 + 4$

$f(x) = (x^2 + 5x + 5)^2 + 4$

$4 \le f(x) \le (36 + 30 + 5)^2 + 4$

$4 \le f(x) \le 5041 + 4 = 5045$

$a + b = 5049.$

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

5. $y = \sin^{-1}[\log_3(x/3)] \Rightarrow -1 \le \log_3(x/3) \le 1$

$\Rightarrow \dfrac{1}{3} \le \dfrac{x}{3} \le 3 \Rightarrow 1 \le x \le 9 \Rightarrow x \in [1, 9]$

6. $f(x) = \log(x + \sqrt{x^2 + 1})$

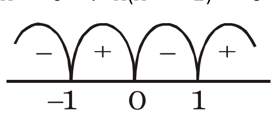
and $f(-x) = -\log(x + \sqrt{x^2 + 1}) = -f(x)$

$f(x)$ is odd function.

7. $f(x) = \dfrac{3}{4 - x^2} + \log_{10}(x^3 - x)$. So, $4 - x^2 \ne 0$

$\Rightarrow x \ne \pm\sqrt{4}$

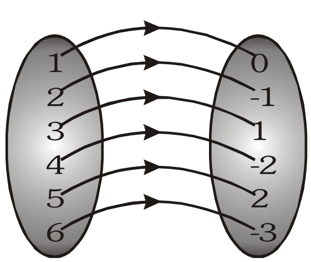
and $x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0 \Rightarrow x > 0, x > 1$



$\therefore D = (-1, 0) \cup (1, \infty) - \{\sqrt{4}\}$ i.e., $D = (-1, 0) \cup (1, 2) \cup (2, \infty)$

9. $f : N \to I$

$f(1) = 0$, $f(2) = -1$, $f(3) = 1$, $f(4) = -2$, $f(5) = 2$ and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

10. To define $f(x)$, $9 - x^2 > 0 \Rightarrow -3 < x < 3$ ... (1)

$-1 \le (x - 3) \le 1 \Rightarrow 2 \le x \le 4$ ... (2)

From (i) and (ii), $2 \le x < 3$ i.e., $[2, 3)$.

11. $f(3) = {}^{7-3}P_0 = 1$, $f(4) = {}^3P_1 = 3$ and $f(5) = {}^2P_2 = 2$

Hence, range of $f = \{1, 2, 3\}$.

12. Using $-\sqrt{a^2 + b^2} \le (a \sin x + b \cos x) \le \sqrt{a^2 + b^2}$

$-\sqrt{1 + (-\sqrt{3})^2} \le (\sin x - \sqrt{3} \cos x) \le \sqrt{1 + (-\sqrt{3})^2}$

$-2 \le (\sin x - \sqrt{3} \cos x) \le 2$

$-2 + 1 \le (\sin x - \sqrt{3} \cos x + 1) \le 2 + 1$

$-1 \le (\sin x - \sqrt{3} \cos x + 1) \le 3$ i.e., range $= [-1, 3]$

$\therefore$ For f to be onto $S = [-1, 3]$.

13. For $-1 < x < 1$, $\tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$

Range of $f(x) = \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ Co-domain of function = B = $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

14. $f(a-(x-a)) = f(a)f(x-a) - f(0)f(x)$ ... (1)

Put x=0, y = 0 ; $f(0) = (f(0))^2 - [f(a)]^2 \Rightarrow f(a) = 0$

$[\because f(0) = 1]$. From (i), $f(2a-x) = -f(x)$.

15. Let $y = \frac{3x^2+9x+17}{3x^2+9x+7}$

$\Rightarrow 3(y-1)x^2 + 9(y-1)x + 7y - 17 = 0$

Since x is real, we have

$\{9(y-1)\}^2 - 4.3(y-1)(7y-17) \geq 0$

$\Rightarrow \quad -3y^2 + 126y - 123 \geq 0$

$\Rightarrow \quad (y-41)(y-1) \leq 0$

$\Rightarrow \quad 1 \leq y \leq 41$

So, maximum value of y is 41.

16. f(x) is defined if $-1 \leq \frac{x}{2} - 1 \leq 1$ and $\cos x > 0$

or $\quad 0 \leq x \leq 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore \quad x \in \left[0, \frac{\pi}{2}\right)$

19. For real x, $f(x) = x^3 + 5x + 1$

$\lim_{x\to\infty} f(x) = +\infty$

and $\quad \lim_{x\to\infty} f(x) = -\infty$

$\therefore$ Range is R – f(x) is on to

Now $f'(x) = 3x^2 + 5 > 0$

$\therefore$ f(x) is one-one

f(x) is one-one onto.

20. $f(x) = (x+1)^2 - 1 \quad ; \quad x \geq -1$

$f'(x) = 2(x+1) \geq 0$ for $x \geq -1$

$\therefore$ f(x) is bijection

Statement (2) is correct

Now $f^{-1}(x) = f(x)$

To solve put y = x in f(x)

$x = (x+1)^2 - 1$

$x + 1 = (x+1)^2$

$x = -1, \quad x = 0$

$x = \{0, -1\}$ Statement (1) is also correct

21. $f(x) = \frac{1}{\sqrt{|x|-x}}$

For domain of real function

$|x| - x > 0$

$|x| > x$

$\boxed{x \in (-\infty, 0)}$

22. $f(x) = (x-1)^2 + 1 \quad ; (x \geq 1)$

and $f'(x) = 2(x-1) \geq 0$ for $x \geq 1$

$\therefore$ $f(x)$ is one-one and onto

$\Rightarrow$ $f(x)$ is Bijection

and $f^{-1}(x) = 1 + \sqrt{x-1}$

Statement-2 is true

Now $\quad f(x) = f^{-1}(x)$

$\Rightarrow (x-1)^2 + 1 = \sqrt{x-1} + 1$

$\Rightarrow x = 1,2$

$\therefore$ Statement-1 is true

23. [x] is contincous at R – I

$\therefore$ f(x) is continuous at R – I

Now At x = I

LHL $= \lim_{h\to 0} [I - h] \cos\frac{(2(I-h)-1)}{2}\pi$

$\lim_{h\to 0} (I-1)\cos[2I-2h-1]\frac{\pi}{2}$

$= (I-1)\cos(2I-1)\frac{\pi}{2} = 0$

similarly,

RHL = 0

and f(I) = 0

$\therefore$ Function is continous everywhere

3. g(x) = 1 + {x}

$\Rightarrow$ 0 + 1 ≤ g(x) ≤ 1 + 1 $\Rightarrow$ 1 ≤ g(x) < 2

f (g(x)) = 1 ($\because$ g(x) > 0)

6. n (into + onto) = 2^4

n (into) = 2

n (onto) = 16 – 2 = 14

7. $f(x) = \dfrac{\alpha x}{x+1}$, $x \neq -1$

Now f (f(x)) = x $\Rightarrow$ f (x) = $f^{-1}(x)$

Let $y = \dfrac{\alpha x}{x+1}$ $\Rightarrow$ xy + y = αx

$\Rightarrow$ x (y – α) = –y $\Rightarrow$ $x = \dfrac{-y}{y-\alpha}$

$f^{-1}(x) = \dfrac{-x}{x-\alpha}$

Now $\dfrac{\alpha x}{x+1} = \dfrac{-x}{x-\alpha}$

on solving we get α = –1

13. ϕ (x) = f (x) – g (x)

$$= \begin{cases} -x & x \in Q \\ x & x \notin Q \end{cases}$$

It is one-one onto function

14. Given $f(x) = x^2$; g(x) = sinx

$f$ogogo$f$(x) = $\sin^2(\sin x^2)$

and gogo$f$(x) = $\sin(\sin x^2)$

given $f$ogogo$f$(x) = gogo$f$(x)

$\Rightarrow$ $\sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow$ $\sin(\sin x^2) = 0$ or 1 (rejected)

$\sin(\sin x^2) = 0$ $\Rightarrow$ $x^2 = n\pi$

$\Rightarrow$ $x = \pm\sqrt{n\pi};\ x \in \{0, 1, 2, 3, ...\}$

15. $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow$ $f'(x) = 6(x^2 - 5x + 6)$

= 6(x – 2)(x – 3)

$\because$ f(x) is non monotonic in x ∈ [0,3]

$\Rightarrow$ f(x) is not one-one

f(x) is increasing in x ∈[0,2) and decreasing in x∈(2,3]

$f(0) = 1$, $f(2) = 29$ & $f(3) = 28$

$\therefore$ Range of f(x) is [1,29]

$\Rightarrow$ f(x) is onto.

16. $\because$ $\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ $\Rightarrow$ $2\theta \in \left(0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right)$

Now $f(\cos 4\theta) = \dfrac{2}{2 - \sec^2\theta}$

$= \dfrac{1 + \cos 2\theta}{\cos 2\theta} = 1 + \dfrac{1}{\cos 2\theta}$ ....(i)

Let $\cos 4\theta = \dfrac{1}{3}$ $\Rightarrow$ $2\cos^2 2\theta - 1 = \dfrac{1}{3}$

$\Rightarrow$ $\cos 2\theta = \pm\sqrt{\dfrac{2}{3}}$

$\Rightarrow$ From (i), $f\left(\dfrac{1}{3}\right) = 1 \pm \sqrt{\dfrac{3}{2}}$

$\Rightarrow$ (A, B) are correct