UNIT # 02

PART-1 : DETERMINANT, MATRIX, TRIGONOMETRIC EQUATION, SOLUTION OF TRIANGLE

DETERMINANT

EXERCISE - 01 CHECK YOUR GRASP

1. Hint : $C_1 \to C_1 + C_2 + C_3$

2. Hint : Put x = 0 on both sides.

7. Applying $R_3 \to R_3 - 3R_1 - 2R_2$ we get $\Delta = 0$

$\Rightarrow$ infinite solution.

8. $a = a_0.r_1^{p-1} \Rightarrow \log a = (p-1)\log r_1 + \log a_0$

$b = a_0.r_1^{q-1} \Rightarrow \log b = (q-1)\log r_1 + \log a_0$

$c = a_0 r_1^{r-1} \Rightarrow \log c = (r-1)\log r_1 + \log a_0$

$$\begin{vmatrix} \log a_0 + (p-1)\log r_1 & p & 1 \\ \log a_0 + (q-1)\log r_1 & q & 1 \\ \log a_0 + (r-1)\log r_1 & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log a_0 & p & 1 \\ \log a_0 & q & 1 \\ \log a_0 & r & 1 \end{vmatrix} + \log r_1 \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix} = 0$$

10. Hint : Put x = 0

11. Do $R_1 \to \log x R_1$, $R_2 \to \log y R_2$, $R_3 \to \log z R_3$

we get $\dfrac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$

12. Hint : $C_1 \to C_1 - C_2$

14. $$\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & (1-x)^2 & 1-2x \\ 2x+1 & 3x & 3x-2 \\ x+1 & 2x & 2x-3 \end{vmatrix}$$

Since two columns are same in above determinants therefore we can add them along C_3.

$$\Rightarrow \begin{vmatrix} (1+x)^2 & (1-x)^2 & -(x+1)^2 \\ 2x+1 & 3x & -(1+2x) \\ x+1 & 2x & -(1+x) \end{vmatrix} = 0$$

$\Rightarrow$ 0 = 0 $\Rightarrow$ infinite solution

16. Hint : $R_3 \to R_1 + R_3$

17. $$D = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ ca^2b^2 & cab & c(a+b) \end{vmatrix}$$

$(R_1 \to aR_1,\ R_2 \to bR_2,\ R_3 \to cR_3)$

$$= abc\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+cb \end{vmatrix}$$

$$= abc(ab+bc+ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} \quad (C_3 \to C_3 + C_1)$$

= 0

21. Hint : $\Delta = 0$

26. $x + y = 3$ ....(i)

$(1+K)x + (K+2)y = 8$ ....(ii)

$x - (1+K)y = -K - 2$ ....(iii)

If system is consistent then $\Delta = 0$

on solving we get

$K = 1, \dfrac{-5}{3}$

EXERCISE - 02 BRAIN TEASERS

2. On performing $R_3 \to R_3 - R_1 - 2R_2$

we get $f'(x) = 0 \Rightarrow f(x) = c$

$\Rightarrow$ a straight line parallel to x-axis

7. On applying $C_1 \to C_1 + C_2 + C_3$

we get $\begin{vmatrix} 1 & x+b^2x & c^2x+x \\ 1 & 1+b^2x & c^2x+x \\ 1 & x+b^2x & c^2x+1 \end{vmatrix} x = f(x)$

Now solve it.

9. $$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2 A & 2\sin 4\theta \\ 2 & 1+\cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 0 & -1 & 0 \\ 2 & 1+\cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow 2(1+2\sin 4\theta) - 2\sin 4\theta = 0$

$\Rightarrow 1 + \sin 4\theta = 0$

$\sin 4\theta = -1$

$4\theta = -\dfrac{\pi}{2}$ or $4\theta = \dfrac{3\pi}{2}$

$\theta = -\dfrac{\pi}{8}$ or $\theta = \dfrac{3\pi}{8}$ & $A \in R$

EXERCISE - 03   MISCELLANEOUS TYPE QUESTIONS

True & False :

1. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= -\frac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 (c - a)^2\}$

3. $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$\Rightarrow \quad (1 - a^2) + c(-c - ab) - b(ac+b) = 0$

$\Rightarrow \quad 1 - a^2 - c^2 - abc - b^2 - abc = 0$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2abc - 1 = 0$

Assertion & Reason :

1. *Statement-I :* $B_1 = c_2a_3 - a_2c_3$

$B_2 = a_1c_3 - c_1a_3, \quad B_3 = a_2c_1 - a_1c_2$

$a_1B_1 = a_1a_3c_2 - a_1a_2c_3$

$a_2B_2 = a_1a_2c_3 - a_2a_3c_1$

$a_3B_3 = a_1a_2c_3 - a_1a_3c_2$

Statement-II is obviously true.

Comprehension # 1 :

Hint :

$\Delta = (x - y)(y - z)(z - x)[xyz(xy + yz + zx) - (x + y + z)]$

Comprehension # 2 :

$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = (\alpha - 1)^2 (\alpha + 2)$

1. $\Delta \neq 0 \Rightarrow$ unique solution

2. $\alpha = -2 \quad \Rightarrow \quad \Delta = 0, \quad \Delta_1 = \begin{vmatrix} m & -2 & 1 \\ n & 1 & -2 \\ p & 1 & 1 \end{vmatrix}$

$\Delta_1 = 3\ (m + n + p) \neq 0 \quad \Rightarrow \quad \Delta_1 \neq 0$

Hence no solution

3. $x + y + z = m$

$x + y + z = p$

$\therefore \quad m \neq p \Rightarrow$ no solution

EXERCISE - 04 [A]   CONCEPTUAL SUBJECTIVE EXERCISE

5. $D' \equiv \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$

On breaking into 8 determinant to we get
$= D + D = 2D$

6. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$C_1 \to C_1 - bC_3\ ,\ C_2 \to C_2 + aC_3$

$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+b^2+a^2) & 1-a^2-b^2 \end{vmatrix}$

$= (1 + a^2 + b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$

$\equiv (1 + a^2 + b^2)^3$

9. **Hint :** On applying $R_1 \to R_1 + R_2 + R_3$
and then solving the determinant we get
$x = 0$ or $x^2 = a^2 + b^2 + c^2 - ab - bc - ca$

$$x = \sqrt{\frac{3}{2}(a^2 + b^2 + c^2)}$$

11. $A28 = 100A + 20 + 8 = k\lambda_1$

$3B9 = 300 + 10B + 9 = k\lambda_2$

$62C = 600 + 20 + C = k\lambda_3$

$\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$

$R_2 \to 100R_1 + 10R_3 + R_2$

$= \begin{vmatrix} A & 3 & 6 \\ A28 & 3B9 & 62C \\ 2 & B & 2 \end{vmatrix} = k \begin{vmatrix} A & 3 & 6 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ 2 & B & 2 \end{vmatrix}$

13. $\sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} 2(3^{r-1}) & \sum_{r=1}^{n} 4(5^{r-1}) \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$

$= \begin{vmatrix} 1+2+\ldots 2^{n-1} & 2\{1+3\ldots 3^{n-1}\} & 4(1+5+\ldots 5^{n-1}) \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$

$$= \begin{vmatrix} \frac{2^n-1}{2-1} & \frac{2(3^n-1)}{3-1} & \frac{4(5^n-1)}{5-1} \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix} = 0$$

14. $D = \frac{1}{x^4z^4}\begin{vmatrix} z & z & -(x+y) \\ -(y+z) & x & x \\ -yz(y+z) & xz(x+2y+z) & -xy(x+y) \end{vmatrix}$

$$= \frac{1}{x^4z^4}\begin{vmatrix} xy & zy & -(x+y)z \\ -x(y+z) & xy & xz \\ -(y+z) & x+2y+z & -(x+y) \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \frac{1}{x^4z^4}\begin{vmatrix} 0 & zy & -(x+y)z \\ 0 & xy & xz \\ 0 & x+2y+z & -(x+y) \end{vmatrix} = 0$$

15. Let $a = \beta + \gamma - \delta - \alpha$, $b = \gamma + \alpha - \beta - \delta$, $c = \alpha + \beta - \gamma - \delta$

we get $\begin{vmatrix} a^4 & a^2 & 1 \\ b^4 & b^2 & 1 \\ c^4 & c^2 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^4-b^4 & a^2-b^2 & 0 \\ b^4-c^4 & b^2-c^2 & 0 \\ c^4 & c^2 & 1 \end{vmatrix}$$

$$= (a^2-b^2)(b^2-c^2)\begin{vmatrix} a^2+b^2 & 1 & 0 \\ b^2+c^2 & 1 & 0 \\ c^4 & c^2 & 1 \end{vmatrix}$$

$= (a^2-b^2)(b^2-c^2)(a^2-c^2)$

$= (a-b)(a+b)(b-c)(b+c)(a+c)(a-c)$

$= -2^6(\alpha-\beta)(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)(\gamma-\delta)$

16. Given determinant split into two given determinant

$$= \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix}\begin{vmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

17. (a) $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 3$

$\Rightarrow D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} = 3$

$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 6$

Similarly $D_3 = 9$

so consistent having $x = 1$, $y = 2$, $z = 3$

(d) $D = \begin{vmatrix} 7 & -7 & 5 \\ 3 & 1 & 5 \\ 2 & 3 & 5 \end{vmatrix} = 0$

$D_1 = \begin{vmatrix} 3 & -7 & 1 \\ 7 & 1 & 1 \\ 5 & 3 & 1 \end{vmatrix} = 24$

Inconsistent system

18. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = \lambda - 3$

$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = \mu - 10$

(a) For unique solution $\lambda \neq 3$

(b) $D = 0$, $D_1 = 0 \Rightarrow \lambda = 3$, $\mu = 10$

(c) $D = 0$, $D_1 \neq 0 \Rightarrow \lambda = 3$, $\mu \neq 10$

20. $\frac{\alpha_1}{\alpha_2} = \frac{\beta_1}{\beta_2} \Rightarrow \frac{(\alpha_1+\alpha_2)^2}{(\alpha_1+\alpha_2)^2 - 4\alpha_1\alpha_2} = \frac{(\beta_1+\beta_2)^2}{(\beta_1+\beta_2)^2 - 4\beta_1\beta_2}$

$\Rightarrow \frac{b^2/a^2}{b^2/a^2 - 4c/a} = \frac{q^2/p^2}{q^2/p^2 - 4r/p}$

$\Rightarrow \frac{b^2}{q^2} = \frac{b^2 - 4ac}{q^2 - 4rp} \Rightarrow \frac{b^2}{q^2} = \frac{4ac}{4rp}$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 9 & 0 \end{vmatrix} = 36$

Aslo $\Delta_u = -12$, $\Delta_v = 24$, $\Delta_w = 60$

So $u = -\frac{1}{3}$, $v = \frac{2}{3}$, $w = \frac{5}{3}$

Also $[(b-c)^2 + (c-a)^2 + (d-b)^2]$

$= d^2 + a^2 + 2ad = (a-d)^2$

So equation is $-\frac{9}{10}x^2 + (a-d)^2x + 2 = 0$

$\Rightarrow -9x^2 + 10(a-d)^2x + 20 = 0$

Put $x = \frac{1}{x}$ we get $20x^2 + 10(a-d)^2x - 9 = 0$

Hence root of given equation are reciprocal to each other.

4. $R_1 \to R_1 - R_2$

$$= 2\begin{vmatrix} \sin x\sin y & \sin y\sin z & \sin x\sin z \\ \cos(x+y) & \cos(y+x) & \cos(z+x) \\ \sin(x+y) & \sin(y+z) & \sin(x+z) \end{vmatrix}$$

$R_2 \to R_1 + R_2$

$$= 2\begin{vmatrix} \sin x\sin y & \sin y\sin z & \sin x\sin z \\ \cos x\cos y & \cos y\cos x & \cos x\cos z \\ \sin(x+y) & \sin(y+z) & \sin(x+z) \end{vmatrix}$$

taking sinxsiny common from C_1, siny sinz from C_2, sinx sinz from C_3.

$= 2\sin^2 x\sin^2 y\sin^2 z$

$$\begin{vmatrix} 1 & 1 & 1 \\ \cot x\cot y & \cot y\cot z & \cot z\cot x \\ \cot x+\cot y & \cot y+\cot z & \cot z+\cot x \end{vmatrix}$$

$$\left[\therefore \frac{\sin(x+y)}{\sin x\sin y} = \cot x + \cot y\right]$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ and expanding

$= 2\sin(x-y)\sin(y-z)\sin(x-z)$

5. $$\begin{vmatrix} 3 & \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 \\ \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 \\ \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 & \alpha^4+\beta^4+\gamma^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} \times \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$

$= (\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$

6. $$\Delta = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$\Delta^2 = \frac{1}{abc}\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \times \begin{vmatrix} ax_1 & by_1 & cz_1 \\ ax_2 & by_2 & cz_3 \\ ax_3 & by_3 & cz_3 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} d & f & f \\ f & d & f \\ f & f & d \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \frac{1}{abc}\begin{vmatrix} d+2f & f & f \\ d+2f & d & f \\ d+2f & f & d \end{vmatrix} = \frac{(d+2f)}{abc}\begin{vmatrix} 1 & f & f \\ 1 & d & f \\ 1 & f & d \end{vmatrix}$$

on solving it we get

$$\Delta^2 = \frac{(d+2f)}{abc}(d-f)^2$$

$$\Delta = (d-f)\left[\frac{d+2f}{abc}\right]^{\frac{1}{2}}$$

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

3. $$\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}(1 + abc) = 0$$

$(a-b)(b-c)(c-a)(1+abc) = 0$

but $a \neq b \neq c$ so $abc = -1$

4. If $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$ are in G.P.

then $\log a_1, \log a_2, \ldots\ldots \log a_n, \ldots\ldots$ are in A.P.

A, A + D, ...

Let common difference of A.P. is D

so $$\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$$

$$\begin{vmatrix} A & A+2D & A+4D \\ A+6D & A+8D & A+10D \\ A+12D & A+14D & A+16D \end{vmatrix}$$

$C_2 \to 2C_2 - (C_1 + C_3)$

$$\begin{vmatrix} A & 0 & A+4D \\ A+6D & 0 & A+10D \\ A+12D & 0 & A+16D \end{vmatrix} = 0$$

5. $a^2 + b^2 + c^2 = -2$

applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$= (1 - x)^2$ so degree is $\rightarrow 2$

6. For no solution

$\Delta - 0$ and Δ_x or Δ_y or Δ_z at least one is not zero.

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

at $\alpha = 1$ their are 3 row are identical so factor of determinant $(\alpha - 1)^2$

and other factor will be find out by $R_1 \rightarrow R_1 + R_3 + R_3$

$\lambda(\alpha + 2)(\alpha - 1)^2 = 0$

$\alpha = -2, 1$

but at $\alpha = 1$

all equation are same so at $\alpha = 1$ system of equation infinite solution and

at $\alpha = -2$

$$\Delta x = \begin{vmatrix} -3 & 1 & 1 \\ -3 & -2 & 1 \\ -3 & 1 & -2 \end{vmatrix}$$

$-3(4 - 1) - 1(6 + 3) + 1(-3 + 6)$

$-9 - 9 + 3 = -15 \neq 0$

so at $\alpha = -2$ system have no solution.

9. $x - cy - bz = 0$

$cx - y + az = 0$ $\quad x \neq 0$; $y \neq 0$, $z \neq 0$

$bx + ay - z = 0$

these system is homogeneous

so $\Delta_x = \Delta_y = \Delta_z = 0$

and at $\Delta = 0 \rightarrow$ system have non zero solution.

$$\Delta = \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$1 - a^2 + c(-c - ab) - b(ac + b) = 0$

$1 - a^2 - b^2 - c^2 - abc - abc = 0$

$a^2 + b^2 + c^2 + 2abc = 1$

12. $\Delta = 0$ (For Non zero solution)

$$\begin{vmatrix} 4 & K & 2 \\ K & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$8 - K(K - 2) + 2(2K - 8) = 0$

$8 - K^2 + 2K + 4K - 16 = 0$

$-K^2 + 6K - 8 = 0$

$K^2 - 6K + 8 = 0$

$(K - 4)(K - 2) = 0$

$K = 2, 4$ Two solution

13. For Trivial sol^n $\Delta \neq 0$

$$\begin{vmatrix} 1 & -K & 1 \\ K & 3 & -K \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$(-3 + K) + K(-K + 3K) + (K - 9) \neq 0$

$2K^2 + 2K - 12 \neq 0$

$K^2 + K - 6 \neq 0$

$(K + 3)(K - 2) \neq 0$

$K \neq -3$

$K \neq 2$

So Ans. $R - \{2, -3\}$

6.(a) Method : 1

$P \equiv (-\sin(\beta - \alpha), -\cos\beta) \equiv (x_1, y_1)$, $Q \equiv (\cos(\beta - \alpha), \sin\beta) \equiv (x_2, y_2)$

and $R \equiv (x_2\cos\theta + x_1\sin\theta,\ y_2\cos\theta + y_1\sin\theta)$

We see that

$$T \equiv \left(\frac{x_2\cos\theta + x_1\sin\theta}{\cos\theta + \sin\theta}, \frac{y_2\cos\theta + y_1\sin\theta}{\cos\theta + \sin\theta}\right) \text{ and}$$

P, Q, T are collinear $\Rightarrow$ P, Q, R are non–collinear

Method : 2

$$\begin{vmatrix} -\sin(\beta-\alpha) & -\cos\beta & 1 \\ \cos(\beta-\alpha) & \sin\beta & 1 \\ \cos(\beta-\alpha+\theta) & \sin(\beta-\theta) & 1 \end{vmatrix}$$

Applying $R_3 \to R_3 + R_1\sin\theta - R_2\cos\theta$

$$\begin{vmatrix} -\sin(\beta-\alpha) & -\cos\beta & 1 \\ \cos(\beta-\alpha) & \sin\beta & 1 \\ 0 & 0 & 1+\sin\theta-\cos\theta \end{vmatrix}$$

$= (1 + \sin\theta - \cos\theta)\ [-\sin\beta \sin(\beta - \alpha) + \cos\beta \cos(\beta - \alpha)]$

$= (1 + \sin\theta - \cos\theta) \cos(2\beta - \alpha) \neq 0$

Hence P, Q, R are non collinear.

(b) $x - 2y + 3z = -1,\ -x + y - 2z = k$ & $x - 3y + 4z = 1$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0 \ \& \ \begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} = 3-k$$

Hence if $k = 3$ then system will have infinite solutions and $k \neq 3$ then system will have no solution. so S(I) & S(II) both are true & (II) is correct explaination for (I).

7. $(y + z) \cos 3\theta = (xyz) \sin 3\theta$ ...(i)

$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z} \quad \text{...(ii)}$$

$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$ ...(iii)

where $yz \neq 0$ and $0 < \theta < \pi$

from (i) & (iii)

$(y + z) \cos 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$

$\Rightarrow$ $z \cos 3\theta + y \sin 3\theta = 0$ ...(iv)

from eq^n (ii)

$2z \cos 3\theta + 2y \sin 3\theta = xyz \sin 3\theta$ ...(v)

from equation (iv) & (v)

$\Rightarrow$ $xyz \sin 3\theta = 0$

$\Rightarrow$ $x \sin 3\theta = 0$ as $yz \neq 0$

Possible cases are either $x = 0$ or $\sin 3\theta = 0$

Case (1) : if $x = 0$

$\Rightarrow$ $y + z = 0 \Rightarrow y = -z$

from eq^n (iv) $\cos 3\theta = \sin 3\theta$

$$\Rightarrow \quad 3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$$

Case (2) : if $\sin 3\theta = 0$

$$\Rightarrow \quad \theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

But these values does not satisfy given equations.

Hence, total number of possible values of θ are 3.