UNIT # 02

PART-2 : SETS, RELATION, MATHEMATICAL REASONING, PMI, STATISTICS

SETS

EXERCISE - I — CHECK YOUR GRASP

1. $A \cap (A \cup B)' = A \cap (A' \cap B')$

$(\because (A \cup B)' = A' \cap B')$

$= (A \cap A') \cap B'$ (by associative law)

$= \phi \cap B'$ $(\because A \cap A' = \phi)$

$= \phi$

2. It is obvious.

4. From De' morgan's law, $(A \cap B)' = A' \cup B'$.

5. $B' = \{1, 2, 3, 4, 5, 8, 9, 10\}$

$\therefore A \cap B' = \{1, 2, 5\} \cap \{1, 2, 3, 4, 5, 8, 9, 10\}$
$= \{1, 2, 5\} = A$

6. Let $x \in A \Rightarrow x \in A \cup B$, $[\because A \subseteq A \cup B]$

$\Rightarrow x \in A \cap B$, $[\because A \cup B = A \cap B]$

$\Rightarrow x \in A$ and $x \in B \Rightarrow x \in B$, $\therefore A \subseteq B$

Similarly, $x \in B \Rightarrow x \in A$, $B \subseteq A$

Now $A \subseteq B$, $B \subseteq A \Rightarrow A = B$.

7. It is obvious.

11. $A \cap B \subseteq A$. Hence $A \cup (A \cap B) = A$.

12. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

15. Null set is the subset of all given sets.

16. Since $\frac{1}{y} \neq 0, \frac{1}{y} \neq 2, \frac{1}{y} \neq \frac{-2}{3}$, $[\because y \in N]$

$\therefore \frac{1}{y}$ can be 1, $[\because y$ can be $1]$

17. It is fundamental concept.

RELATIONS

EXERCISE - I — CHECK YOUR GRASP

2. Since $x \not< x$, therefore R is not reflexive. Also $x < y$ does not imply that $y < x$, So R is not symmetric. Let xRy and yRz. Then, $x < y$ and $y < z \Rightarrow x < z$ i.e., xRz. Hence R is transitive.

3. For any $x \in R$, we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number.

$\Rightarrow$ xRx for all x. So, R is reflexive.

R is not symmetric, because $\sqrt{2}\,R\,1$ and $1 \not R \sqrt{2}$,

R is not transitive also because $\sqrt{2}\,R\,1$ and $1\,R\,2\sqrt{2}$ but $2 \not R\, 2\sqrt{2}$.

4. R_4 is not a relation from X to Y, because $(7, 9) \in R_4$ but $(7, 9) \notin X \quad Y$.

5. Here $\alpha R \beta \Leftrightarrow \alpha \perp \beta \therefore \alpha \perp \beta \Leftrightarrow \beta \perp \alpha$

Hence, R is symmetric.

7. It is obvious.

13. We have (a, b) R (a, b) for all $(a, b) \in N \quad N$

Since $a + b = b + a$. Hence, R is reflexive.

R is symmetric for we have $(a,b)R(c,d) \Rightarrow a+d=b+c$

$\Rightarrow d + a = b + c \Rightarrow c + b = d + a \Rightarrow (c,d)\,R(a,b)$

Hence R is symmetric

Then by definition of R, we have

$a + d = b + c$ and $c + f = d + e$,

hence by addition, we get

$a + d + c + f = b + c + d + e$ or $a + f = b + e$

Hence, (a, b) R (e, f)

Thus, $(a, b)\,R(c, d)$ and $(c, d)\,R(e, f) \Rightarrow (a, b)\,R(e, f)$.

Hence R is transitive.

14. For $(a, b), (c, d) \in N \quad N$

$(a, b)\,R(c, d) \Rightarrow ad(b + c) = bc(a + d)$

Reflexive : Since $ab(b + a) = ba(a + b)\ \forall ab \in N$,

$\therefore (a, b)R(a, b)$, $\therefore$ R is reflexive.

Symmetric : For $(a, b), (c, d) \in N \quad N$, let $(a,b)R(c,d)$

$\therefore ad\,(b + c) = bc(a + d) \Rightarrow bc(a + d) = ad(b + c)$

$\Rightarrow cb(d + a) = da(c + b) \Rightarrow (c, d)R(a, b)$

$\therefore$ R is symmetric

Transitive : For $(a, b), (c, d), (e, f) \in N \quad N$, Let (a, b)R(c, d), (c, d)R(e, f)

$\therefore$ ad(b + c) = bc(a + d), cf(d + e) = de(c + f)

$\Rightarrow$ adb + adc = bca + bcd ... (i)

and cfd + cfe = dec + de$f$ ... (ii)

(i) ef + (ii) ab gives,

adbe$f$ + adce$f$ + c$f$dab + ce$f$ab

= bcae$f$ + bcde$f$ + decab + de$f$ab

$\Rightarrow$ adc$f$(b + e) = bcde(a + f)

$\Rightarrow$ af(b + e) = be(a + f)

$\Rightarrow$ (a, b)R(e, f).

$\therefore$ R is transitive. Hence R is an equivalence relation

15. Here R is a relation A to B defined by ' x is greater then y'

$\therefore$ R = {(2, 1) ; (3, 1)}. Hence, range of R = {1}.

16. Here $\ell_1 R \ell_2$, ℓ_1 is parallel to ℓ_2 and $\ell_2$is parallel to ℓ_1, so it is symmetric.

Clearly, it is also reflexive and transitive. Hence it is equivalence relation.

20. Let (a, b) $\in$ R

Then, (a, b) $\in$ R $\Rightarrow$ (b,a) $\in$ R^{-1}, [by def or R^{-1}]

$\Rightarrow$ (b, a) $\in$ R, [$\because$ R = R^{-1}], So R is symmetric.

23. R is reflexive if it contains (1, 1) (2, 2) (3, 3)

$\because$ (1, 2) $\in$ R, (2, 3) $\in$ R

$\therefore$ R is symmetric if (2, 1), (3, 2) $\in$ R

Now, R = {(1, 1), (2, 2),(3, 3),(2, 1),(3, 2),(2, 3), (1, 2)}

R will be transitive if (3, 1) ; (1, 3) $\in$ R. Thus, R becomes and equivalence relation by adding (1, 1) (2, 2) (3, 3) (2, 1), (3, 2), (1, 3), (1, 2). Hence, the total number of ordered pairs is 7.

24. Obviously, the relation is not reflexive and transitive but it is symmetric, because

$$x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$$

25. Clearly, the relation is symmetric but it is neither reflexive nor transitive.

26. It is obvious

27. We have, R = {(1,3); (1,5); (2,3); (2,5); (3,5); (4,5)}

R^{-1} = {(3, 1); (5, 1); (3, 2); (5, 2); (5, 3); (5, 4)}

Hence ROR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}

28. It is obvious

29. Given R, and S are relations on set A.

$\therefore$ R $\subseteq$ A A and S $\subseteq$ A A $\Rightarrow$ R $\cap$ S $\subseteq$ A A

$\Rightarrow$ R $\cap$ S is also a relation on A.

Reflexivity : Let a be an arbitrary element of A. Then a $\in$ A $\Rightarrow$ (a, a) $\in$ R [$\because$ R and S are reflexive] and (a, a) $\in$ S

$\Rightarrow$ (a, a) $\in$ R $\cap$ S

Thus, (a, a) $\in$ R $\cap$ S for all a $\in$ A.

So, R $\cap$ S is a reflexive relation on A.

Symmetry : Let a, b $\in$ A such that (a, b)$\in$ R $\cap$ S.

Then, (a, b) $\in$ R $\cap$ S $\Rightarrow$ (a, b) $\in$ R and (a, b) $\in$ S

$\Rightarrow$ (b, a) $\in$ R and (b, a) $\in$ S

[$\because$ R and S are symmetric]

$\Rightarrow$ (b, a) $\in$ R $\cap$ S

Thus, (a, b) $\in$ R $\cap$ S $\Rightarrow$ (b, a) $\in$ R $\cap$ S for all (a, b) $\in$ R $\cap$ S.

So, R $\cap$ S is symmetric on A.

Transitivity : Let a,b,c $\in$ A such that (a,b)$\in$ R $\cap$ S and (b, c) $\in$ R $\cap$ S. Then (a, b) $\in$ R $\cap$ S and (b, c) $\in$ R $\cap$ S

$\Rightarrow$ {((a, b) $\in$ R and (a, b) $\in$ S)}

and {((b, c) $\in$ R and (b, c) $\in$ S)}

$\Rightarrow$ {(a, b) $\in$ R, (b, c) $\in$ R} and {(a,b) $\in$ S, (b, c)$\in$S}

$\Rightarrow$ (a, c) $\in$ R and (a, c) $\in$ S

$$\left[\begin{array}{l}\because \text{R and S transitive, SO} \\ (a,b) \in R \text{ and } (b,c) \in R \Rightarrow (a,c) \in R \\ (a,b) \in S \text{ and } (b,c) \in S \Rightarrow (a,c) \in S\end{array}\right.$$

$\Rightarrow$ (a, c) $\in$ R $\cap$ S

Thus, (a, b) $\in$ R $\cap$ S and (b, c) $\in$ R $\cap$ S

$\Rightarrow$ (a, c) $\in$ R $\cap$ S. So R $\cap$ S is transitive on A

Hence, R is an equivalence relation on A.

EXERCISE - II — PREVIOUS YEAR QUESTION

1. Given A = {1, 2, 3, 4}

R = {(1, 3), (4, 2), (2, 4), (2, 3) (3, 1)}

(2, 3) $\in$ R but (3, 2) $\notin$ R. Hence R is not symmetric.

R is not reflexive as (1, 1) $\notin$ R.

R is not a function as (2, 4) $\in$ R and (2, 3) $\in$ R.

R is not transitive as (1, 3) $\in$ R and (3, 1) $\in$ R but (1, 1) $\notin$ R.

2. Here (3, 3), (6, 6), (9, 9), (12, 12), [Reflexive];

(3, 6), (6, 12), (3, 12), [Transitive].

Hence, reflexive and transitive only.

3. Relation R = {(x, y) $\in$ W   W | the words x and y have at least one letter in common}

R is reflexive as every word has the same letters with itself.

R is symmetric also

But R is not transitive

For example, BOLD is related BAT

BAT is releated to APE

But BOLD has no letter in common with APE.

4. For R, xRy $\Rightarrow$ x = wy

For reflexive

xRx $\Rightarrow$ x = wx

Which is true then w = 1

For symmetric

consider x = 0, y $\neq$ 0

xRy $\Rightarrow$ 0Ry $\Rightarrow$ 0 = wy

which is true when w = 0

Now

yRx $\Rightarrow$ yR0 $\Rightarrow$ y = w   0

There is no rational value of w

for which y = w   0

Hence relation is not symmetric and hence not an equivalence relation

<u>Now for S</u>

For reflexive

$\frac{m}{n}$ S $\frac{m}{n}$ $\Rightarrow$ mn = nm

which is true

For symmetric

Let $\frac{m}{n}$ S $\frac{m}{n}$ $\Rightarrow$ qm = np

$\frac{p}{q}$ S $\frac{m}{n}$ $\Rightarrow$ pn = mq

which is true

Relation is symmetric

For transitive

Let $\frac{m}{n}$ S $\frac{p}{q}$ $\Rightarrow$ qm = pn ... (1)

$\frac{p}{q}$ S $\frac{r}{s}$ $\Rightarrow$ ps = rq ... (2)

From Equation (1) and equation (2)

$\Rightarrow$ ms = nr

$\therefore$ $\frac{m}{n}$ S $\frac{r}{s}$

S is transitive

$\therefore$ S is equivalence.